

# Statistical Significance

MGMT 662: Integrative Research Project

July 24, 2008.

## 1

### 1.1 Goals

#### Goals of this class meeting

- Learn how to interpret something as “statistically significant”
- Learn about the behavior of statistics obtained from samples.
- Learn about the particular behavior of the sample mean, and the importance of the normal distribution.

## 2 Sampling distributions

### 2.1 Definition

#### Sampling distribution

- Imagine taking a sample of size 100 from a population and computing some kind of statistic.
- The statistic you compute can be anything, such as: mean, median, mode, standard deviation, variance, range, a weighted mean, or anything else you might imagine.
- Suppose you repeated this experiment over and over: take a sample of 100 and compute and record the statistic.
- A **sampling distribution** is the probability distribution *of the statistic*
  - Is this the same thing as the probability distribution of the population?
  - The re-sampling experiment is assumed to be done *with replacement* to assure the samples are independent.

### Example

- [Sampling Distribution Simulator](#)
- The sampling distribution is a hypothetical distribution. Why does it matter?

## 2.2 Desirable qualities

### Desirable qualities

What are some qualities you would like to see in a sampling distribution?

- The average of the sample statistics is equal to the true population parameter.

$$\mu_{\bar{x}} = \mu$$

- Want the variance of *the sampling distribution* to be as small as possible. Why?
- Want the *sampling distribution* to be normal, regardless of the distribution of the population.

## 2.3 Central Limit Theorem

### Central Limit Theorem

- Given:
  - Suppose a RV  $x$  has a distribution (it need not be normal) with mean  $\mu$  and standard deviation  $\sigma$ .
  - Suppose a sample mean is computed from a sample of size  $n$ .
- Then, if  $n$  is sufficiently large, the sampling distribution of  $\bar{x}$  will have the following properties:
  - The sampling distribution of  $\bar{x}$  will be normal.
  - The mean of the sampling distribution will equal the mean of the population (consistent):
$$\mu_{\bar{x}} = \mu$$
  - The standard deviation of the sampling distribution will decrease with larger sample sizes, and is given by:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

**Central Limit Theorem: Small samples**

If  $n$  is small (rule of thumb:  $n < 30$ )

- The sample mean is still consistent.
- Sampling distribution will be normal if the distribution of the population is normal.

**2.4 Examples****Example 1**

Suppose average birth weight is  $\mu = 7lbs$ , and the standard deviation is  $\sigma = 1.5lbs$ .

What is the probability that a sample of size  $n = 30$  will have a mean of  $7.5lbs$  or greater?

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$
$$z = \frac{7.5 - 7}{1.5/\sqrt{30}} = 1.826$$

The probability the sample mean is greater than  $7.5lbs$  is:

$$P(\bar{x} > 7.5) = P(z > 1.826) = 0.0339$$

**Example 2**

Suppose average birth weight is  $\mu = 7lbs$ , and the standard deviation is  $\sigma = 1.5lbs$ .

What is the probability that a randomly selected baby will have a weight of  $7.5lbs$  or more? What do you need to assume to answer this question? Must assume the population is normally distributed. Why?

$$z = \frac{\bar{x} - \mu}{\sigma} = \frac{7.5 - 7}{1.5} = 0.33$$

The probability that a baby is greater than  $7.5lbs$  is:

$$P(x > 7.5) = P(z > 0.33) = 0.3707$$

### Example 3

- Suppose average birth weight of all babies is  $\mu = 7lbs$ , and the standard deviation is  $\sigma = 1.5lbs$ .
- Suppose you collect a sample of 30 newborn babies whose mothers used illegal drugs during pregnancy.
- Suppose you obtained a sample mean  $\bar{x} = 6lbs$ . If you assume the mean birth weight of babies whose mothers used illegal drugs has the same sampling distribution as the rest of the population, what is the probability of getting a sample mean this low or lower?

### Example 3 continued

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$
$$z = \frac{6 - 7}{1.5/\sqrt{30}} = -3.65$$

The probability the sample mean is less than or equal to  $6lbs$  is:

$$P(\bar{x} < 6) = P(z < -3.65) = 0.000131$$

That is, if using drugs during pregnancy actually does still lead to an average birth weight of 7 pounds, there was only a 0.000131 (or 0.0131%) chance of getting a sample mean as low as six or lower. This is an extremely unlikely event if the assumption is true. Therefore it is likely the assumption is not true.

## 3 Hypotheses Tests

### 3.1 Constructing hypotheses

#### Statistical Hypotheses

- A **hypothesis** is a claim or statement about a property of a population.
  - Example: The population mean for systolic blood pressure is 120.
- A **hypothesis test** (or **test of significance**) is a standard procedure for testing a claim about a property of a population.
- Recall the example from last week about birth weights with mothers who use drugs.
  - Hypothesis: Using drugs during pregnancy leads to an average birth weight of 7 pounds (the same as with mothers who do not use drugs).

## Null and Alternative Hypotheses

- The **null hypothesis** is a statement that the value of a population parameter (such as the population mean) *is equal to* some claimed value.
  - $H_0: \mu = 7$ .
- The **alternative hypothesis** is an alternative to the null hypothesis; a statement that says a parameter differs from the value given in the null hypothesis.
  - $H_a: \mu < 7$ .
  - $H_a: \mu > 7$ .
  - $H_a: \mu \neq 7$ .
- In hypothesis testing, assume the null hypothesis is true until there is strong statistical evidence to suggest the alternative hypothesis.
- Similar to an “innocent until proven guilty” policy.

## 3.2 Test Statistics

### Hypothesis tests

- (Many) hypothesis tests are all the same:

$$z \text{ or } t = \frac{\text{sample statistic} - \text{null hypothesis value}}{\text{standard deviation of the sampling distribution}}$$

- Example: hypothesis testing about  $\mu$ :
  - Sample statistic =  $\bar{x}$ .
  - Standard deviation of the sampling distribution of  $\bar{x}$ :

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

- Another example: hypothesis testing about  $\mu_1 - \mu_2$ , when each population has the same variance:
  - Sample statistic =  $\bar{x}_1 - \bar{x}_2$ .
  - Standard deviation of the sampling distribution of  $\bar{x}_1 - \bar{x}_2$ :

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\left( \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \right) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$$

### 3.3 P-values

#### P-values

- The P-value of the test statistic, is the area of the sampling distribution from the sample result in the direction of the alternative hypothesis.
- Interpretation: If the null hypothesis is correct, than the p-value is the probability of obtaining a sample that yielded your statistic, or a statistic that provides even stronger evidence of the null hypothesis.
- The p-value is therefore a measure of *statistical significance*.
  - If p-values are very small, there is strong statistical evidence in favor of the alternative hypothesis.
  - If p-values are large, there is insignificant statistical evidence. When large, you fail to reject the null hypothesis.
- Best practice is writing research: report the p-value. Different readers may have different opinions about how small a p-value should be before saying your results are statistically significant.