Probability

Mgmt 230: Introductory Statistics

Goals of this section

- Learn basics of probability.
- Learn about how a boring and complicated formula can really mean a lot.

1 Basic Probability

1.1 Probability of Events

Basic Probability

- **Probability:** numeric value between 0 and 1 (or 0% and 100%) representing the chance, likelihood, or possibility some event will occur.
- **Event:** some possible (or even impossible) outcome occurring.
  - Denote events with capital English letters.
- Example:
  - A: A newborn baby will be female.
  - $P(A) = 0.5$ means there is a 50% chance that a newborn baby is female.
- Computing probability:

$$P(A) = \frac{n(A)}{T}$$

  - $n(A)$ = number of ways event A can occur.
  - T = total number of possible outcomes.

Contingency Table

	Purchased DVR		
TV Purchased	Yes	No	Total
HDTV	38	42	80
Regular TV	70	150	220
Total	108	192	300

- Define Event A: Purchased an HDTV.
- What is $P(A)$?

Joint Events

- **Joint Event:** is an event that is composed of two or more events.
- Define event C as any event in either A *or* B.
  - Notation for event C: $C = A \cup B$
  - Notation for probability of event C: $P(C) = P(A \cup B)$.
- Define event C as any event in A *and* B.
  - Notation for event C: $C = A \cap B$
  - Notation for probability of event C: $P(C) = P(A \cap B)$.

Complements of Events

- The **complement** of an event, A, is the outcome of anything *besides* A occurring.
- Notation: $A' = A^c$ = complement of event A.
- $P(A') = 1 - P(A)$.
- Example: what is the complement of Event A: a newborn baby is a female.

Mutually Exclusive Events

- Two events, A and B, are **mutually exclusive** if it is impossible for both A and B to occur at the same time.
- Are the following mutually exclusive?
  - Event A: A person is currently 8 years old. Event B: A person voted for Kerry in the last presidential election.
  - Event A: A person plays football in high school. Event B: A person plays basketball in high school.
  - Event A, Event A'.
  - Event A: We will go out to eat tonight. Event B: We are going out to eat tomorrow night.

Contingency Table

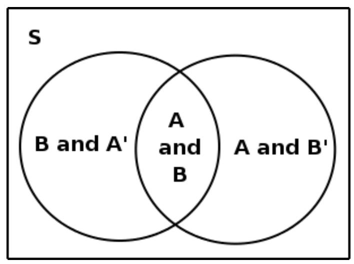
	Purchased DVR		
TV Purchased	Yes	No	Total
HDTV	38	42	80
Regular TV	70	150	220
Total	108	192	300

- Define Event A: Purchased an HDTV.

- Define Event B: Purchased a DVR.
- Define Event $C = A \cap B$.
- What is $P(C)$?
- Define Event $D = A \cup B$.
- What is $P(D)$?

1.2 Venn Diagram

Venn Diagram



- **Venn diagram:** visualization of all possible events. The areas in the diagram represent the probabilities of those events.
- S: Event that encompasses all possible outcomes.
- Entire area of the left hand circle is $P(B)$.
- Entire area of the right hand circle is event $P(A)$.
- The area that is in both of the circles is $P(A \cap B)$.

Venn Diagram

- From the Venn Diagram we can see that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Use this equation to find the probability of owning an HDTV (Event A) or owning a DVR (Event B).

	Purchased DVR		
TV Purchased	Yes	No	Total
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Total	108	192	300

2 Conditional Probability

2.1 What is Conditional Probability

Conditional probability

- **Conditional probability**: the probability of an event, A, with the additional inflation that some other event B has already occurred.
- Example:
  - What is the probability of being female?
    Define event A: a person is female.
    $P(A) = 0.5$
  - What is the probability of being female, given you are a nurse?
    Define event B: a person in a nurse.
    $P(A|B) = 0.8$ (I just made that up)

2.2 Independence

Independence

- Two events A and B are **independent** if knowledge that A happened does not affect P(B), or if knowledge that B happened does not effect P(A).
- In the example above, is being female and being a nurse independent?
- More examples:
  - Is the event that someone smokes and the event someone has lung cancer independent?
  - Suppose a coin is flipped twice. Is the event the first flip is heads and the event the second flip is heads independent?
- If A and B are independent, then
  $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

2.3 Bayes Theorem

Bayes Theorem

- This is the coolest thing you'll ever learn a math class:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Why is the cool? Because this proves that:

$$P(A|B) \neq P(B|A)$$

Bayes Theorem

- Not so cool example: Suppose $P(A) = 0.4$, $P(B) = 0.8$, and $P(A \cap B) = 0.2$. What is $P(A|B)$?

$$P(A|B) = \frac{0.2}{0.8} = 0.25$$

- Are these events independent?

Blood test accuracy

- Suppose a fatal disease breaks out, and a blood test is used to detect the disease.
- The blood test claims to accurately identify the disease 99% of the time.
- Let A be the event you have a disease.
- Let B be the event the blood test came out positive.

$$P(B|A) = 0.99$$

- Suppose you take the blood test and it is positive. What is the probability you have the disease?

$$P(A|B) = \ ?$$

Blood test accuracy

- Suppose 0.2% of people have the disease, and 0.198% have the disease and tested positive.

$$P(A) = 0.002$$

$$P(A \cap B) = 0.00198$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.00198}{0.002} = 0.99$$

Blood test accuracy

- Suppose 5% of people test positive for the disease.
- What is the probability you have the disease given you tested positive?

$$P(A|B) = \frac{P(A \cap B)}{P(A)} = \frac{0.00198}{0.05} = 0.0396$$

- Even though you tested positive, *you still most likely do not have the disease.*
- And the test had the claim of being 99% accurate.
- So what is the real accuracy of things like blood tests, pregnancy tests, and lie detector tests?

3

Homework

- Probability basics: Section 4.1, page 156, problems 4.3 through 4.10.
- Conditional probability: Section 4.2, page 164, problems 4.16 through 4.24.