Confidence Interval for a Mean

Math 130: Introductory Statistics

Goals of this class meeting

- Learn how to estimate a population mean, and report a margin of error.
- Learn what a confidence interval is, and how to interpret it.

Margin of error

- **Point estimate**: a single value used to approximate a population parameter.
    - Example: a sample mean is a point estimate for the population mean.
- **Margin of error**: the maximum likely distance of a point estimate from the actual population parameter.
- The margin of error depends on the desired **level of confidence**.
    - The larger the level of confidence, the larger the margin of error.
    - Example: estimate today's high temperature.

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Confidence interval

- **Confidence interval**: a range of values (instead of a single point estimate) used to estimate a population parameter.
- The width of the confidence interval depends on the **level of confidence**.
- **Level of confidence**: a probability given by, $1 - \alpha$. It is equal to the proportion of times that similarly constructed confidence intervals will contain the true population parameter.
- Let E denote the margin of error for a sample mean, $\bar{x}$. The confidence interval is simply,

$$(\bar{x} - E, \bar{x} + E)$$

- Example, say $\bar{x} = 40$ and the margin of error for a 95% is 15. Then the *95% confidence interval is* $(25, 55)$.

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Interpretation of CI

- Previous example: The 95% confidence interval for $\bar{x}$ is $(25, 55)$.
- Correct interpretation: 95% of similarly constructed confidence intervals will contain the true population mean.
- That is, if you were to repeat the experiment many, many times, and compute each confidence interval, 95% of these confidence intervals will contain the true mean.
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Computing confidence intervals

- For now, let's make the stupid assumption we know the population parameter, σ.
- The margin of error is given by:

$$E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

- $z_{\alpha/2}$ is called a **critical value**. The larger the level of confidence, the larger is $z_{\alpha/2}$.
- Therefore the confidence interval is:

$$(\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}})$$

- This method of computing confidence intervals depends on the central limit theorem. So it is only applicable if the assumptions of the central limit theorem are met.

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Critical values

- We know from the Central Limit Theorem that the sampling distribution for $\bar{x}$ is normal and has a standard deviation of $\frac{\sigma}{\sqrt{n}}$.
- Therefore, if we want a 95% confidence interval, find values for $\bar{x}$ such that 95% of the sampling distribution will be within that range.
- What values for z include 95% of the normal distribution?
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Desired samples sizes

- Suppose before you conducted a study or gathered any data, you have a goal that the margin of error not be larger than some size.
- You need to know how much data to collect to make that happen.

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2$$

- E is the largest margin of error you will tolerate, $1 - \alpha$ is the confidence level of that margin of error.
- You will need to have *at least* this many observations or else your margin of error will be larger than E.

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Examples

- Suppose you take a sample of 65 Viterbo graduates and find the mean income is $37,600. It is known the population standard deviation is $3,500. Compute a 95% confidence interval for mean income of Viterbo graduates.
  - First: find critical z-values.
  - Plug and chug into the formula:

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$E = 1.96\frac{3500}{\sqrt{65}} = 850.88$$

$$(37600 - 850.88, 37600 + 850.88)$$

$$(36749.12, 38450.88)$$

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Examples

- Use same example. Compute a 90% confidence interval. Is the range larger or smaller?
- Compute a 95% confidence interval, but suppose the sample size is 30. Is the range larger or smaller?
- Suppose you desire a 95% margin of error not to exceed $700. What sample size must be collected to make this happen?
- Suppose you desire a 99% margin of error not to exceed $700. What sample size must be collected to make this happen?
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- More uncertainty means we need to widen the confidence interval.
- Use the T-statistic (instead of using Z).
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