

Introduction to Management Science

Management 560: Management Science

Tuesday, March 10, 2009

1 Plan

Learning Objective	Active Learning Activity
Understand what management science is.	Lecture / Discussion
Learn how to build mathematical models from real world type decision problems.	Worksheet problem.
Learn how to solve break-even models.	Worksheet problem.
Learn how to solve profit maximization models.	Worksheet problem.
Practice makes perfect!	Another worksheet problem.
Learn linear programming	Lecture/Discussion Two worksheet problems.
Assess what we have learned.	Quiz.

1.1 Goals

Goals of this class meeting

- Learn what is management science.
- Learn two basic techniques for modeling decision problems.
- Learn some basic quantitative techniques for solving such problems.

2 Model Building

Management Science

- A.k.a. Operations Research, Quantitative Analysis, Decision Analysis, Decision Sciences, but strangely enough, not Operations Management.
- Application of quantitative methods to help managers make decisions that achieve one or more stated objectives.
- **Model:** abstract mathematical representation of a managers objective, possible decisions, and constraints.

2.1 Handcrafted Dolls

Model: Example

Andy Mendoza makes handcrafted dolls, which he sells at craft fairs, and is considering mass-producing the dolls to sell in stores. He estimates that the initial investment for the plant and equipment will be \$10,000, the cost of labor will be \$5 per doll, the cost of material will be \$2 per doll, the cost of packaging and shipping will be \$3 per doll, and Andy will be able to sell the dolls for \$20 each.

Worksheet questions

1. What might be Andy's objective if he's hoping to make it rich in the handmade dolls game?
2. What might be Andy's objective if he plans on this being a non-profit charitable organization that hires people less fortunate.
3. What **variable(s)** does Andy have influence over? That is what can he make decisions on?

Looking out for #1

Andy is sick of helping ungrateful people, and from here on out Andy only wants to worry about Andy. Write down his objective function now. Does this problem seem realistic? Why not?

It turns out dolls won't just sell for \$20 no matter what; he can increase or decrease the price of dolls, changing how many dolls people are willing to buy. Suppose for each \$1 increase in price, the quantity of dolls demanded drops by 40, and if Andy raises his price to \$50, then he will not be able to sell any.

Worksheet questions

1. Andy now has two decision variables (though one depends on the other). What are they?
2. What is Andy's new constraint?
3. Plug in the constraint into Andy's objective function. This will result in a *quadratic objective function* that has a finite maximum.
4. Calculus tells us that a quadratic function of the form $f(x) = ax^2 + bx + c$ reaches a maximum or minimum when you set $x = -b/2a$. Figure out how much Andy should produce and what price to charge. How do we know if we minimized or maximized Andy's profit?

2.2 Retread Tire Company

Retread Tire Company: Breaking Even

The Retread Tire Company recaps tires. The fixed annual cost of the recapping operation is \$60,000. The **variable cost** of recapping a tire is \$9 per tire. Suppose the company charges \$25 to recap a tire, and the company is interested in breaking even.

Worksheet questions

1. What is the company's objective?
2. What are the company's decision variables?
3. What is the company's objective function?
4. For an annual volume of 12,000 tires, determine the total cost, total revenue, and total profit.
5. Determine the break even volume for the Retread Tire Company.

Retread Tire Company: Maximizing Profit

Suppose instead of being able to charge \$25 per fixed tire, the demand for tire fixes decreases by 50 sales for each \$1 increase in price. Suppose also that if the price for tire fixes were as high as \$80, Retread Tire Company would not find a single customer.

Worksheet questions

1. What are the company's decision variables?
2. What is the company's objective function?
3. What is the company's demand constraint?
4. Plug the demand constraint into the objective function to obtain the quadratic objective function. What is optimal choice for quantity and price?

3 Linear Programming

3.1 Objectives and Constraints

Linear Programming

- Linear programming: problems that involve *linear* objective functions and *linear* constraints.
- Possible objectives:
 - maximize profits, minimize costs, maximize revenues, minimize time to complete projects, minimize the total distances needed to transport goods from warehouses to stores, minimize total energy consumption.
- Objective function is subject to constraints such as:
 - Limited resources, restrictive guidelines, demand for goods at stores, supply of goods at warehouses, energy requirements for machines or buildings, time requirements for completing intermediate steps of a project.

3.2 Assembly Lines

Assembly Lines

Suppose a company produces two products that are processed on two assembly lines. Assembly line 1 has 100 available hours, and assembly line 2 has 42 available hours. Each product requires 10 hours on assembly line 1, while Product A needs 7 hours on assembly line 2, and Product B needs 3 hours on assembly line 2. The company earn \$6 profit for every item of Product A produced and \$4 profit for every item of Product B produced.

Worksheet questions

1. What is the company's objective?
2. What are the company's choice variables? Denote these x_1 , x_2 , etc.
3. What is the company's objective function?
4. What are the company's constraints? What's stopping them from producing infinite amounts of of each product? Write down the functional relationships for the constraints. Did you use equal signs or inequalities? Which is appropriate?
5. Graph the objective function if profit is equal to 30 (this could be called an *iso-profit function*).

6. On the same graph, plot the iso-profit function for profit equal to 40.
7. On the same graph, plot the iso-profit function for profit equal to 50.
8. What do you notice about the graphs?

9. On a new graph, plot the constraints and shade the **feasibility region**.
The feasibility region is the set of all decisions that satisfy all constraints.

10. Understanding the behavior of iso-profit functions, show on your graph the possible candidates for the decision variables that maximize the company's objective?
11. Compute exactly the quantities for the decision variables.

12. What decision maximizes the company's objective?

3.3 Copperfield Mining Company

Copperfield Mining Company

Copperfield Mining Company owns two mines, each of which produces three grades of ore - high, medium, and low. The company has a contract to supply a smelting company with at least 12 tons of high-grade ore, 8 tons of medium-grade ore, and 24 tons of low-grade ore. Mine 1 produces 6 tons of high-grade ore, 2 tons of medium-grade ore, and 4 tons of low-grade ore for each hour it operates. Mine 2 produces 2 tons of high-grade ore, 2 tons of medium-grade ore, and 4 tons of low-grade ore for each hour it operates. The cost of operating mines is \$200 for mine 1 and \$160 for mine 2.

Worksheet questions

1. What is the company's objective?
2. What are the company's choice variables? Denote these x_1 , x_2 , etc.
3. What is the company's objective function?
4. What are the company's constraints? What's stopping them from producing infinite amounts of of each product? Write down the functional relationships for the constraints.
5. Plot the constraints and shade the **feasibility region**. The feasibility region is the set of all decisions that satisfy all constraints.

6. Compute all possible candidates for the optimal decision variables.

7. What decision maximizes the company's objective?

4

Homework

- Modeling basics and breaking even:
 - Pages 23-24, problems 2, 3, 15.
 - “Ocobee River Rafting Company” case problem, page 27.
 - “Constructing a Downtown Parking Lot in Draper” case problem, page 28.
- Linear programming:
 - Page 61, problem 5.

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2. Suppose instead demand for rugs varies depending on price. If the price is \$200, Ray will be able to sell 150 rugs per month, and for every \$1 increase in price, monthly sales decrease by 2.

(a) Write down an equation for the quantity of rugs sold in terms of price.

(b) Write down an equation for the total cost.

(c) Write down an equation for the total revenue as a function of price.

(d) What is Ray's objective function if his objective is to maximize profits?

(e) What is Ray's profit maximizing choice for price and quantity?