

Linear Programming

Management 560: Management Science

Tuesday, March 17, 2009

Goals of this class meeting

- Learn how to set up problems with constraints.
- Learn how to maximize or minimize objectives subject to constraints.
- Learn how to solve linear programming problems in Excel.

Linear Programming

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- Linear programming: problems that involve *linear* objective functions and *linear* constraints.
- Possible objectives:
 - maximize profits, minimize costs, maximize revenues, minimize time to complete projects, minimize the total distances needed to transport goods from warehouses to stores, minimize total energy consumption.
- Objective function is subject to constraints such as:
 - Limited resources, restrictive guidelines, demand for goods at stores, supply of goods at warehouses, energy requirements for machines or buildings, time requirements for completing intermediate steps of a project.

Assembly Lines

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Suppose a company produces two products that are processed on two assembly lines. Assembly line 1 has 100 available hours, and assembly line 2 has 42 available hours. Each product requires 10 hours on assembly line 1, while Product A needs 7 hours on assembly line 2, and Product B needs 3 hours on assembly line 2. The company earn \$6 profit for every item of Product A produced and \$4 profit for every item of Product B produced.

Worksheet questions

Copperfield Mining Company

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Copperfield Mining Company owns two mines, each of which produces three grades of ore - high, medium, and low. The company has a contract to supply a smelting company with at least 12 tons of high-grade ore, 8 tons of medium-grade ore, and 24 tons of low-grade ore. Mine 1 produces 6 tons of high-grade ore, 2 tons of medium-grade ore, and 4 tons of low-grade ore for each hour it operates. Mine 2 produces 2 tons of high-grade ore, 2 tons of medium-grade ore, and 4 tons of low-grade ore for each hour it operates. The cost of operating mines is \$200 for mine 1 and \$160 for mine 2.

Worksheet questions

Shadow Prices

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- **Shadow Prices:** aka “dual prices”, aka “marginal values”, aka “Lagrange multipliers”, is the price the decision maker would be willing to pay for a marginal ease in one of the constraints.
- Example from Assembly lines: what is the maximum price the assembly line manager be willing to pay for one extra allowable hour for Assembly line A?
- Example from Mining Company: how much would the mining company save if it could produce one less ton of low-grade iron ore?
- Example from Farmers on quiz: how much would the farmer be willing to pay for one additional hour of labor?
- Be careful with shadow prices: they assume one and only one change is happening. They are not valid with more than one change.

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Bluegrass Distillery

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Bluegrass Distillery produces a custom-blend whiskey made from rye and bourbon. The company has received an order for a minimum of 400 gallons. Manufacturing constraints limit the distillery from making more than 500 gallons. The customer has specified that the bourbon should contain at least 40% rye and not more than 250 gallons of bourbon. The blend is sold for \$5 per gallon, and the company's costs are \$2 per gallon of rye and \$1 per gallon of bourbon. The company wants to determine a mix that will maximize its profits.

Worksheet questions

- Linear programming:
 - Chapter 2, page 69, “Metropolitan Police Patrol” case problem.
 - Chapter 2, page 70, “Annabelle Invests in the Market”.
 - Chapter 3, page 105, problem #47, parts a, b, and c.