Network Models

Management 560: Management Science

March 31, 2009

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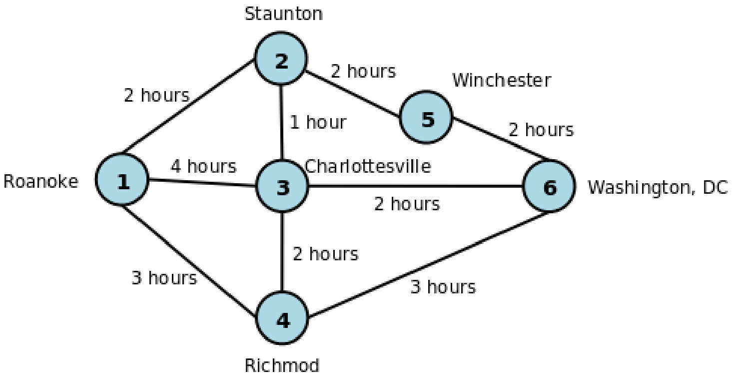
Example

- Goal: Drive from Roanoke, VA to Washington DC, using the fastest route (without using Mapquest).
- The possible routes and distances are given below.



- Start with source, add it to the **permanent set**.
- Find closest node to the permanent set, add it to the permanent set.
- Add this node to the permanent set, and note the distance from the source.
- Repeat: add the node that is adjacent to the permanent set, that is closest to the source.
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- Design a binary linear programming problem, where each branch is a decision variable.
- In Excel, make a row for each branch, leave a blank column for the decision, and enter a column of travel times.
- From this, you can compute the total travel time.
- Constraints:
  - Source node: all branches out must add to 1.
  - Destination node: all branches in must add to 1.
  - Intermediate nodes: all branches in must equal all branches out.
- Figure out the optimal path and driving time for the example problem.

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Minimal Spanning Tree

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- Start at any node.
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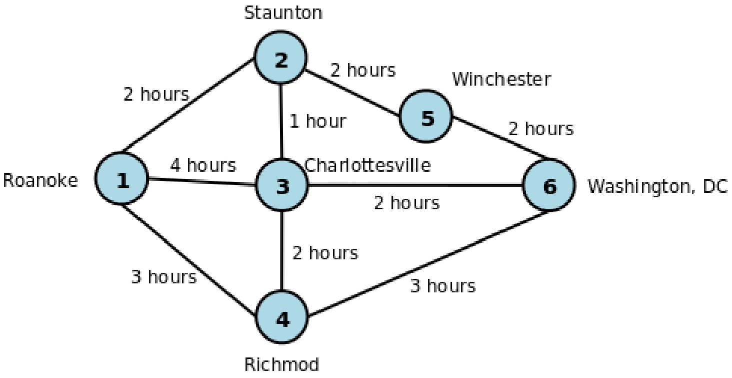
- The possible routes and distances are given below.



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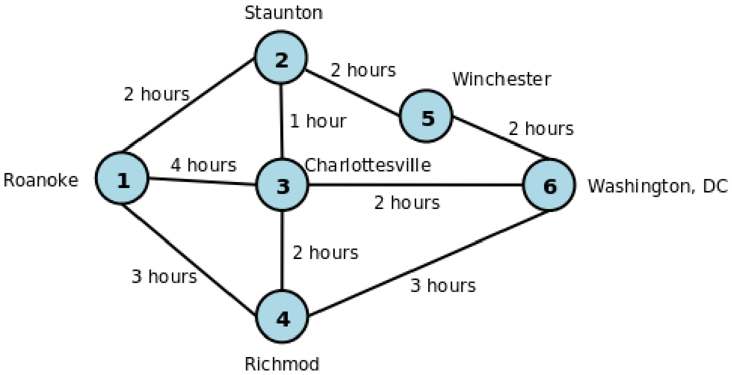

- In previous problems: the branches did not have a limited capacity.
- Objective: select a single path that maximizes the total amount of flow from the source to the destination.
- Problem: paths are limited by the amount of flow they will allow.
- Examples:
    - Maximize the amount of flow of oil, gas, or water through pipelines.
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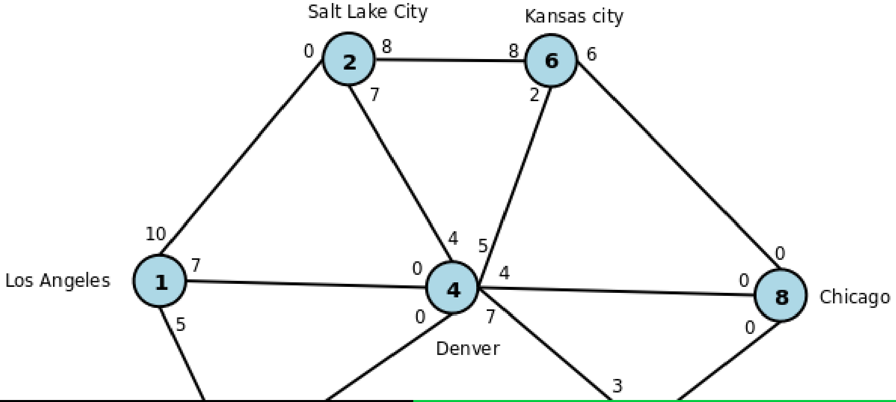
Suppose the FAA has just granted a license to a new airline. The airline wants to maximize the number of flights it can send from Los Angeles to Chicago, but for safety reasons, FAA regulations restrict the amount of air traffic between the cities. The flights per day for each route are shown in the following network:



- Notice traffic can move in *two directions*: each unique path and direction is a decision variable.
- If there are n nodes, there are a maximum of $n^2 + 2$ variables and even more constraints.
- Create two extra variables for the total coming in to the network, and the total leaving the network.
- Each path, in each direction, has its own constraint.
- The total flow of traffic leaving the source must equal the total flow entering destination.
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Homework!

- Chapter 7, page 306, problem 23.
- Chapter 7, page 311-312, problems 36, 37 (this one will be presented for a case problem).