

Univariate and Bivariate Tests

BUS 230: Business and Economics Research and Communication

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1.1 Goals

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- Specific goals:
 - Be able to distinguish different types of data and prescribe appropriate statistical methods.
 - Conduct a number of hypothesis tests using methods appropriate for questions involving only one or two variables.
- Learning objectives:
 - LO2: Interpret data using statistical analysis.
 - LO2.3: Formulate conclusions and recommendations based upon statistical results.

1.2 Hypotheses Tests

Statistical Hypotheses

- A **hypothesis** is a claim or statement about a property of a population.
 - Example: The population mean for systolic blood pressure is 120.
- A **hypothesis test** (or **test of significance**) is a standard procedure for testing a claim about a property of a population.
- Recall the example about birth weights with mothers who use drugs.
 - Hypothesis: Using drugs during pregnancy leads to an average birth weight of 7 pounds (the same as with mothers who do not use drugs).

Null and Alternative Hypotheses

- The **null hypothesis** is a statement that the value of a population parameter (such as the population mean) *is equal to* some claimed value.
 - $H_0: \mu = 7$.
- The **alternative hypothesis** is an alternative to the null hypothesis; a statement that says a parameter differs from the value given in the null hypothesis.
- Pick *only one* of the following for your alternative hypothesis. Which one depends on your research question.
 - $H_a: \mu < 7$.
 - $H_a: \mu > 7$.
 - $H_a: \mu \neq 7$.
- In hypothesis testing, assume the null hypothesis is true until there is strong statistical evidence to suggest the alternative hypothesis.
- Similar to an “innocent until proven guilty” policy.

Hypothesis tests

- (Many) hypothesis tests are all the same:

$$z \text{ or } t = \frac{\text{sample statistic} - \text{null hypothesis value}}{\text{standard deviation of the sampling distribution}}$$

- Example: hypothesis testing about μ :
 - Sample statistic = \bar{x} .
 - Standard deviation of the sampling distribution of \bar{x} :

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

P-values

- The P-value of the test statistic, is the area of the sampling distribution from the sample result in the direction of the alternative hypothesis.
- Interpretation: If the null hypothesis is correct, than the p-value is the probability of obtaining a sample that yielded your statistic, or a statistic that provides even stronger evidence of the null hypothesis.
- The p-value is therefore a measure of *statistical significance*.

- If p-values are very small, there is strong statistical evidence in favor of the alternative hypothesis.
- If p-values are large, there is insignificant statistical evidence. When large, you fail to reject the null hypothesis.
- Best practice is writing research: report the p-value. Different readers may have different opinions about how small a p-value should be before saying your results are statistically significant.

2 Univariate Tests

2.1 Types of Data/Tests

Types of Data

- Nominal data: consists of categories that cannot be ordered in a meaningful way.
- Ordinal data: order is meaningful, but not the distances between data values.
 - Excellent, Very good, Good, Poor, Very poor.
- Interval data: order is meaningful, *and* distances are meaningful. However, there is *no natural zero*.
 - Examples: temperature, time.
- Ratio data: order, differences, and zero are all meaningful.
 - Examples: weight, prices, speed.

Types of Tests

- Different types of data require different statistical methods.
- Why? With interval data and below, operations like addition, subtraction, multiplication, and division are *meaningless!*
- Parametric statistics:
 - Typically take advantage of central limit theorem (imposes requirements on probability distributions)
 - Appropriate only for interval and ratio data.
 - More **powerful** than nonparametric methods.
- Nonparametric statistics:

- Do not require assumptions concerning the probability distribution for the population.
- There are many methods appropriate for ordinal data, some methods appropriate for nominal data.
- Computations typically make use of data's *ranks* instead of actual data.

2.2 Hypothesis Testing about Mean

Single Mean T-Test

- Test whether the population mean is equal or different to some value.
- Uses the sample mean its statistic.
- T-test is used instead of Z-test for reasons you may have learned in a statistics class. Interpretation is the same.
- Parametric test that depends on results from Central Limit Theorem.
- Hypotheses
 - Null: The population mean is equal to some specified value.
 - Alternative: The population mean is [greater/less/different] than the value in the null.

Example Questions

- Suppose we have a dataset of individual schools and the average pay for teachers in each school, and the level of spending as a ratio of the number of students (spending per pupil).
 - Show some descriptive statistics for teacher pay and expenditure per pupil.
 - Is there statistical evidence that teachers make less than \$50,000 per year?
 - Is there statistical evidence that expenditure per pupil is more than \$7,500?

Example: Public School Spending

- Dataset: average pay for public school teachers and average public school spending per pupil for each state and the District of Columbia in 1985.
- Download dataset `eduspending.sav`.
- Conduct the following exercises:

- Show some descriptive statistics for teacher pay and expenditure per pupil.
- Is there statistical evidence that teachers make less than \$25,000 per year?
- Is there statistical evidence that expenditure per pupil is more than \$3,500?

2.3 Hypothesis Testing about Proportion

Single Proportion T-Test

- **Proportion:** Percentage of times some characteristic occurs.
- Example: percentage of consumers of soda who prefer Pepsi over Coke.

$$\text{Sample proportion} = \frac{\text{Number of items that has characteristic}}{\text{sample size}}$$

- Example questions:
 - Are more than 50% of potential voters most likely to vote for Barack Obama in the next presidential election?
 - Suppose typical brand-loyalty turn-over in the mobile phone industry is 10%. Is there statistical evidence that AT&T has brand-loyalty turnover more than 10%?

Example: Economic Outlook

- Data from Montana residents in 1992 concerning their outlook for the economy.
- All data is ordinal or nominal:
 - AGE = 1 under 35, 2 35-54, 3 55 and over
 - SEX = 0 male, 1 female
 - INC = yearly income: 1 under \$20K, 2 20-35\$K, 3 over \$35K
 - POL = 1 Democrat, 2 Independent, 3 Republican
 - AREA = 1 Western, 2 Northeastern, 3 Southeastern Montana
 - FIN = Financial status 1 worse, 2 same, 3 better than a year ago
 - STAT = 0, State economic outlook better, 1 not better than a year ago
- Do the majority of Montana residents feel their financial status is the same or better than one year ago?
- Do the majority of Montana residents have a more positive economic outlook than one year ago?

2.4 Nonparametric Testing about Median

Single Median Nonparametric Test

- Why?
 - Ordinal data: cannot compute sample means (they are meaningless), only median is meaningful.
 - Small sample size and you are not sure the population is not normal.
- Sign test: can use tests for proportions for testing the median.
 - For a null hypothesized population median...
 - Count how many observations are above the median.
 - Test whether that proportion is greater, less than, or not equal to 0.5.
 - For small sample sizes, use binomial distribution instead of normal distribution.

Example Questions

- Instructor evaluations have an ordinal scale: Excellent, Very Good, Good, Poor, Very Poor.
 - Is there statistical evidence that the median rating for a professor is below 'Very Good'?
- Suppose you have a frequency determination question on your survey. Is this an ordinal scale? Is the median an appropriate measure of center?

Example: Attitudes Grade School Kids

- Dataset: 438 students in grades 4 through 6 were sampled from three school districts in Michigan. Students ranked from 1 (most important) to 5 (least important) how important grades, sports, being good looking, and having lots of money were to each of them.
- Open dataset `gradeschools.sav`. Choose second worksheet, titled `Data`.
- Answer some of these questions:
 - Is the median importance for grades is greater than 3?
 - Is the median importance for money less than 3?

3 Bivariate Tests

3.1 Difference in Populations (Independent Samples)

Difference in Means (Independent Samples)

- Suppose you want to know whether the mean from one population is larger than the mean for another.
- Independent samples means you have different individuals in your two sample groups.
- Examples:
 - Compare sales volume for stores that advertise versus those that do not.
 - Compare production volume for employees that have completed some type of training versus those who have not.
- Statistic: Difference in the sample means ($\bar{x}_1 - \bar{x}_2$).
- Hypotheses:
 - Null hypothesis: the difference between the two means is zero.
 - Alternative hypothesis: the difference is [above/below/not equal] to zero.

Example

- Dataset: average pay for public school teachers and average public school spending per pupil for each state and the District of Columbia in 1985.
- Test the following hypotheses:
 - Does spending per pupil differ in the North (region 1) and the South (region 2)?
 - Does teacher salary differ in the North and the West (region 3)?
- Do you see any weaknesses in our statistical analysis?

Nonparametric Tests for Differences in Medians

- Mann-Whitney U test: nonparametric test to determine difference in *medians*.
- Can you suggest some examples?
- Assumptions:
 - Samples are independent of one another.

- The underlying distributions have the same shape (i.e. only the location of the distribution is different).
- It has been argued that violating the second assumption does not severely change the sampling distribution of the Mann-Whitney U test.
- Null hypothesis: medians for the two populations are the same.
- Alternative hypotheses: medians for the two populations are different.

3.2 Paired Samples

Dependent Samples - Paired Samples

- Use a **paired sampled test** if instead the two samples have the same individuals before and after some treatment.
- Examples:
 - The Biggest Loser: Compare the weight of people on the show before the season begins and one year after the show concludes.
 - Training session: Are workers more productive 6 months after they attended some training session versus before the training session.
- Really simple: for each individual subtract the before treatment measure from the after treatment measure (or vice-versa).
- Treat your new series as a single series.
- Conduct one-sample tests.

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Conclusions

- Ideas to keep in mind:
 - What is a sampling distribution? What does it imply about p-values and statistical significance?
 - When it is appropriate to use parametric versus non-parametric methods.
 - Most univariate and bivariate questions have a parametric and non-parametric approach.
- Next class: closed-book, closed note quiz (so study!); and an in-class exercise practicing this stuff in SPSS.
- Next: Statistics to understand the relationship between two variables.