Statistical Significance and Univariate and Bivariate Tests

BUS 230: Business and Economics Research and Communication

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1.1 Goals

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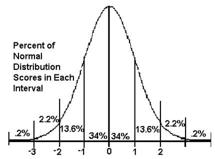
- Specific goals:
  - Re-familiarize ourselves with basic statistics ideas: sampling distributions, hypothesis tests, p-values.
  - Be able to distinguish different types of data and prescribe appropriate statistical methods.
  - Conduct a number of hypothesis tests using methods appropriate for questions involving only one or two variables.
- Learning objectives:
  - LO2: Interpret data using statistical analysis.
  - LO2.3: Formulate conclusions and recommendations based upon statistical results.

2 Statistical Significance

2.1 Sampling Distribution

Probability Distribution

- **Probability distribution:** summary of all possible values a variable can take along with the probabilities in which they occur.
- Usually displayed as:

Picture 

Table

z	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531

Formula

$$f(x|\mu,\sigma) =$$

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

- **Normal distribution:** often used "bell shaped curve", reveals probabilities based on how many standard deviations away an event is from the mean.

Sampling distribution

- Imagine taking a sample of size 100 from a population and computing some kind of statistic.
- The statistic you compute can be anything, such as: mean, median, proportion, difference between two sample means, standard deviation, variance, or anything else you might imagine.
- Suppose you repeated this experiment over and over: take a sample of 100 and compute and record the statistic.
- A **sampling distribution** is the probability distribution *of the statistic*
- Is this the same thing as the probability distribution of the population?

Example

- Sampling Distribution Simulator
- In reality, you only do an experiment once, so the sampling distribution is a hypothetical distribution.
- Why are we interested in this?

Desirable qualities

What are some qualities you would like to see in a sampling distribution?

- The average of the sample statistics is equal to the true population parameter.
- Want the variance *of the sampling distribution* to be as small as possible. Why?
- Want the *sampling distribution* to be normal, regardless of the distribution of the population.

2.2 Central Limit Theorem

Central Limit Theorem

- Given:
  - Suppose a RV x has a distribution (it need not be normal) with mean μ and standard deviation σ.
  - Suppose a *sample mean* ($\bar{x}$) is computed from a sample of size n.
- Then, if n is sufficiently large, the sampling distribution of $\bar{x}$ will have the following properties:
  - The sampling distribution of $\bar{x}$ will be normal.
  - The mean of the sampling distribution will equal the mean of the population (unbiased):
  $$\mu_{\bar{x}} = \mu$$
  - The standard deviation of the sampling distribution will decrease with larger sample sizes, and is given by:
  $$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Central Limit Theorem: Small samples

If n is small (rule of thumb for a single variable: $n < 30$)

- The sample mean is still unbiased.
- The formula for the standard deviation of the sampling distribution still holds ($\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$), but with a small n, the sampling distribution may be wide.
- Sampling distribution will be normal *only if* the distribution of the population is normal, so using the central limit theorem requires this additional assumption.

Example 1

Suppose average birth weight is $\mu = 7lbs$, and the standard deviation is $\sigma = 1.5lbs$.

What is the probability that a sample of size $n = 30$ will have a mean of $7.5lbs$ or greater?

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$z = \frac{7.5 - 7}{1.5/\sqrt{30}} = 1.826$$

In Excel, we can compute $P(z > 1.826)$ using the formula
`=1 - normsdist(1.826)`. The probability the sample mean is greater than $7.5lbs$ is:

$$P(\bar{x} > 7.5) = P(z > 1.826) = 0.0339$$

Example 2

Suppose average birth weight is $\mu = 7lbs$, and the standard deviation is $\sigma = 1.5lbs$.

What is the probability that a randomly selected baby will have a weight of 7.5lbs or more? What do you need to assume to answer this question?

Must assume the population is normally distributed. Why?

$$z = \frac{x - \mu}{\sigma} = \frac{7.5 - 7}{1.5} = 0.33$$

In Excel, we can compute $P(z > 0.33)$ using the formula
`=1 - normsdist(0.33)`. The probability that a baby is greater than 7.5lbs is:

$$P(x > 7.5) = P(z > 0.33) = 0.3707$$

Example 3

- Suppose average birth weight of all babies is $\mu = 7lbs$, and the standard deviation is $\sigma = 1.5lbs$.
- Suppose you collect a sample of 30 newborn babies whose mothers smoked during pregnancy.
- Suppose you obtained a sample mean $\bar{x} = 5\ lbs$. If you assume the mean birth weight of babies whose mothers smoked during pregnancy has the same sampling distribution as the rest of the population, what is the probability of getting a sample mean this low?

Example 3 continued

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$z = \frac{5 - 7}{1.5/\sqrt{30}} = -7.30$$

In Excel, we can compute $P(z < -7.30)$ using the formula
`=normsdist(-7.30)`.

The probability the sample mean is less than or equal to $5lbs$ is:

$$P(\bar{x} < 5) = P(z < -7.30) = 0.00000000000014$$

That is, if smoking during pregnancy actually truly lead to an average birth weight of 7 pounds (we began with this assumption), there was only a 0.00000000000014 (or 0.000000000014%) chance of getting a sample mean as low as six or lower.

This is an extremely unlikely event if the assumption is true. Therefore it is likely the assumption is not true.

2.3 Hypotheses Tests

Statistical Hypotheses

- A **hypothesis** is a claim or statement about a property of a population.
  - Example: The population mean for income per household in the United States is $45,000.
- A **hypothesis test** (or **test of significance**) is a standard procedure for testing a claim about a property of a population.
- Recall the example about birth weights with mothers who use drugs.
  - Hypothesis: Smoking during pregnancy leads to an average birth weight of 7 pounds (the same as with mothers who do not use drugs).

Null and Alternative Hypotheses

- The **null hypothesis** is a statement that the value of a population parameter (such as the population mean) **is equal to** some value.
  - H_0: $\mu = 7$.
- The **alternative hypothesis** is an alternative to the null hypothesis; a statement that says a parameter **is different than** the value given in the null hypothesis.
- Pick *only one* of the following for your alternative hypothesis. Which one depends on your research question.
  - H_a: $\mu < 7$.
  - H_a: $\mu > 7$.
  - H_a: $\mu \neq 7$.
- In hypothesis testing, assume the null hypothesis is true until there is strong statistical evidence to suggest the alternative hypothesis.
- Similar to an "innocent until proven guilty" policy.

Hypothesis tests

- (Many) hypothesis tests are all the same:

$$z \text{ or } t = \frac{\text{sample statistic} - \text{null hypothesis value}}{\text{standard deviation of the sampling distribution}}$$

- Example: hypothesis testing about μ:
  - Sample statistic $= \bar{x}$.
  - Standard deviation of the sampling distribution of $\bar{x}$:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

P-values

- The P-value of the test statistic, is the area of the sampling distribution from the sample result in the direction of the alternative hypothesis.
- Interpretation: If the null hypothesis is correct, than the p-value is the probability of obtaining a sample that yielded your statistic, or a statistic that provides even stronger evidence against the null hypothesis.
- The p-value is therefore a measure of **statistical significance**.
  - If p-values are very small, there is strong statistical evidence in favor of the alternative hypothesis.
  - If p-values are large, there is insignificant statistical evidence. When large, you fail to reject the null hypothesis.
- **Significance level:** often denoted by α, a threshold p-value for deciding to reject versus fail to reject a null hypothesis.
- Common significance levels: $\alpha = 0.05$, $\alpha = 0.1$, $\alpha = 0.01$.
- Best practice is writing research: report the p-value. Different readers may have different opinions about how small a p-value should be before saying your results are statistically significant.

3 Univariate Tests

3.1 Types of Data/Tests

Types of Data

- **Nominal data:** consists of categories that cannot be ordered in a meaningful way.
- **Ordinal data:** order is meaningful, but not the distances between data values.
  - Excellent, Very good, Good, Poor, Very poor.
- **Interval data:** order is meaningful, *and* distances are meaningful. However, there is *no natural zero.*
  - Examples: temperature, time.
- **Ratio data:** order, differences, and zero are all meaningful.
  - Examples: weight, prices, speed.
  - Special example: **binary data:** observations that are all equal to either 0 or 1, indicating whether or not some characteristic exists.

Types of Tests

- Different types of data require different statistical methods.
- Why? With interval data and below, operations like addition, subtraction, multiplication, and division are *meaningless!*
- Parametric statistics:
  - Typically take advantage of central limit theorem (imposes requirements on sample size and/or probability distribution for the population)
  - Appropriate only for interval and ratio data.
- Nonparametric statistics:
  - Do not require the same assumptions concerning the probability distribution for the population.
  - There are many methods appropriate for ordinal data, some methods appropriate for nominal data.

Deciding on a Statistical Test

When deciding what statistical test to use for your research question and data, ask yourslef the following questions:

1. Always keep in mind, what is your research question. What did you measure?
2. How many variables did you measure?
3. What is the scale of measurement? Nominal / Ordinal / Interval / Ratio
4. If you have two or more measurements, are you looking for a difference or another relationship?
5. If you are looking for a difference, are your measurements independent or paired?

3.2 Hypothesis Testing about Mean

Single Mean T-Test

- Test whether the population mean is equal or different to some value.
- Uses the sample mean its statistic.
- T-test is used instead of Z-test for reasons you may have learned in a statistics class. Interpretation is the same.
- Parametric test that depends on results from Central Limit Theorem.

- Hypotheses
  - Null: The population mean is equal to some specified value.
  - Alternative: The population mean is [greater/less/different] than the value in the null.

Example Questions

- Suppose we have a dataset of individual schools and the average pay for teachers in each school, and the level of spending as a ratio of the number of students (spending per pupil).
  - Show some descriptive statistics for teacher pay and expenditure per pupil.
  - Is there statistical evidence that teachers make less than $50,000 per year?
  - Is there statistical evidence that expenditure per pupil is more than $7,500?

3.3 Hypothesis Testing about Proportion

Single Proportion T-Test

- **Proportion:** Percentage of times some characteristic occurs.
- Example: percentage of consumers of soda who prefer Pepsi over Coke.

$$\text{Sample proportion} = \frac{\text{Number of items that has characteristic}}{\text{sample size}}$$

- Example questions:
  - Are more than 50% of potential voters most likely to vote for Barack Obama in the next presidential election?
  - Suppose typical brand-loyalty turn-over in the mobile phone industry is 10%. Is there statistical evidence that AT&T has brand-loyalty turnover more than 10%?
- You can alternatively just use a single mean test for a proportion, where the variable is binary (0,1) and can be treated as interval/ratio data.

3.4 Nonparametric Testing about Median

Single Median Nonparametric Test

- Why?

  - Ordinal data: cannot compute sample means (they are meaningless), only median is meaningful.
  - Small sample size and you are not sure the population is not normal.

- Sign test: can use tests for proportions for testing the median.
  - For a null hypothesized population median...
  - Count how many observations are above the median.
  - Test whether that proportion is greater, less than, or not equal to 0.5.
  - For small sample sizes, use binomial distribution instead of normal distribution.

Example Questions

- Instructor evaluations have an ordinal scale: Excellent, Very Good, Good, Poor, Very Poor.
  - Is there statistical evidence that the median rating for a professor is below 'Very Good'?
- Suppose you have a frequency determination question on your survey. Is this an ordinal scale? Is the median an appropriate measure of center?

4 Bivariate Tests

4.1 Difference in Populations (Independent Samples)

Difference in Means (Independent Samples)

- Suppose you want to know whether the mean from one population is larger than the mean for another.
- Independent samples means you have different individuals in your two sample groups.
- Examples:
  - Compare sales volume for stores that advertise versus those that do not.
  - Compare production volume for employees that have completed some type of training versus those who have not.
- Statistic: Difference in the sample means ($\bar{x}_1 - \bar{x}_2$).
- Hypotheses:
  - Null hypothesis: the difference between the two means is zero.
  - Alternative hypothesis: the difference is [above/below/not equal] to zero.

Nonparametric Tests for Differences in Medians

- Mann-Whitney U test: nonparametric test to determine difference in *medians.*
- Can you suggest some examples?
- Assumptions:
  - Samples are independent of one another (different individuals or sampling-units in each group).
  - The underlying distributions have the same shape (i.e. only the median of the distribution is different).
  - It has been argued that violating the second assumption does not severely change the sampling distribution of the Mann-Whitney U test.
- Null hypothesis: medians for the two populations are the same.
- Alternative hypotheses: medians for the two populations are different.

4.2 Paired Samples

Dependent Samples - Paired Samples

- Use a **paired sampled test** if the two samples have the same individuals or sampling units.
- Many examples include before/after tests for differences:
  - The Biggest Loser: Compare the weight of people on the show before the season begins and one year after the show concludes.
  - Training session: Are workers more productive 6 months after they attended some training session versus before the training session.
- Examples besides before/after tests for differences:
  - Do students spend more time studying than watching TV?
  - Does the unemployment rate for White/Caucasian differ from the unemployment rate for African Americans (sampling unit = U.S. state).
- These are not independent samples, because you have the same individuals in each group.

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Conclusions

- Ideas to keep in mind:
  - What is a sampling distribution? What does it imply about p-values and statistical significance?
  - When it is appropriate to use parametric versus non-parametric methods.
  - Most univariate and bivariate questions have a parametric and non-parametric approach.
  - Decision Tree
- Next class: In-class exercise practicing this stuff in SPSS.
- Next: Statistics to understand *relationships* or *comovements* between two variables.