# Logistic Regression

BUS 735: Business Decision Making and Research

## 1

**Goals of this section**

- Specific goals:
  - Learn how to conduct regression analysis with a dummy independent variable.
- Learning objectives:
  - LO2: Be able to construct and use multiple regression models (including some limited dependent variable models) to construct and test hypotheses considering complex relationships among multiple variables.
  - LO6: Be able to use standard computer packages such as SPSS and Excel to conduct the quantitative analyses described in the learning objectives above.
  - LO7: Have a sound familiarity of various statistical and quantitative methods in order to be able to approach a business decision problem and be able to select appropriate methods to answer the question.

**Logistic Regression**

- **Logistic Regression**: method for estimating a regression with a *dummy dependent variable*.
- Will a potential customer purchase a product?(YES=1, NO=0).
  - Might use explanatory variables: age, gender, income, etc.
- Will a potential employee be retained after one year?(YES=1, NO=0).
  - Might use explanatory variables: age, gender, years experience, past income, education dummy (4-year = 1, otherwise = 0).
- Why not run a regression? Which assumption is violated?

**Structural Form**

- Normal regression:

$$y_i = b_0 + b_1 X_{1,i} + b_2 X_{2,i} + ... + b_{k-1} X_{k-1,i} + e_i$$

- Logistic regression:

$$\log(Odds) = b_0 + b_1 X_{1,i} + b_2 X_{2,i} + ... + b_{k-1} X_{k-1,i} + e_i$$

$$Odds = \frac{P(y_i = 1)}{1 - P(y_i = 1)} = \frac{P(y_i = 1)}{P(y_i = 0)}$$

- Remember, $y_i \in 0, 1$ indicates YES=1 event did occur, or NO=0 event did not occur.

**Predicted Value**

- Predicted value from a regular regression:

$$\hat{y}_i = b_0 + b_1 X_{1,i} + b_2 X_{2,i} + ... + b_{k-1} X_{k-1,i}$$

- For a logistic regression, you can get predicted logist (not too interesting yet):

$$\hat{l}_i = ln(\widehat{Odds}) = b_0 + b_1 X_{1,i} + b_2 X_{2,i} + ... + b_{k-1} X_{k-1,i}$$

- To uncover the **predicted probability of the event occuring:**

$$P(\widehat{y_i = 1}) = \frac{1}{1 + e^{(-\hat{l}_i)}}$$

**Marginal Effects**

- **Marginal effect for regression:** measure of how much $y$ changes when $x$ increases by 1.
  - Example: How much does public expenditure per capita increase (or decrease) when economic ability increases by one unit?
- **Marginal effect for logit:** measure of how much $P(y_i = 1)$ changes when $x$ increases by 1.
  - How much more (or less) likely will an interview candicate be working here after one year if she/he has a four year college education?

**Coefficients versus Marginal Effects**

- For a regular regression, for coefficient $b_2$:
  - The sign (positive/negative) indicates whether $x_2$ causes $y$ to increase or decrease.
  - The magnitude tells *how much* $y$ increases when increasing $x_2$ by 1.
  - Coefficient = Marginal Effect.
- For a logistic regression, the coefficient $b_2$:
  - The sign (positive/negative) indicates whether $x_2$ causes $y$ to increase or decrease.
  - The magnitude is pretty meaningless.
  - Need to do more to figure out marginal effect.
- Because SPSS is stupid, it cannot figure out marginal effects.
- For a specific individual (specific values for X's), compute predicted probabilities by hand for:
  - The individual's actual set of values for X's.
  - Same set of X's, except increase one of them by 1.
  - Take the difference = Marginal effect.

**Example**

- Well Being Dataset (Described on pages 356-357 of SAB).
- 182 college students in Washington, D.C. area.
- Variables of interest:
  - Weight (4 categories: UNDERWEIGHT=1, NORMAL=2, OVERWEIGHT=3, OBESE=4)
  - Gender (2 categories: MALE=0, FEMALE=1)
  - Age
  - Race (AFRICAN AMERICAN=1, WHITE=2, OTHER=3)
  - Marital Status (SINGLE=1, MARRIED=2, OTHER=3)
  - Financial Status (OVEREXTENDED=1, MAKING ENDS MEET=2, COMFORTABLE=3)
  - Physical Health (5 point Likert scale)
  - Depression Scale (0-60, values $> 15$ indicate depression)
- Find preliminary evidence whether each variable is related to weight.
- Logistic Regression Analysis.