Introduction to Probability

BUS 735: Business Decision Methods and Research

Goals of this section

- Learn basics of probability.
- Learn about how a boring and complicated formula can really mean a lot.

Basic Probability

- **Probability:** numeric value between 0 and 1 (or 0% and 100%) representing the chance, likelihood, or possibility some event will occur.
- **Event:** some possible (or even impossible) outcome occurring.
  - Denote events with capital English letters.
- Example:
  - A: A new restaurant will earn positive profits during its first year.
  - $P(A) = 0.2$ means there is a 20% chance that a new restaurant will earn profits in its first year.
- **A-priori probability:** objective probability of an event that can be stated *before* the event occurs.
- **Relative frequency probability:** long-run expected probabilities based on past data.
- **Subjective probability:** estimated probability based on personal belief, experience, or knowledge of a situation.

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Contingency Table

	Customer Owns a DVR		
Type of TV	Yes	No	Total
HDTV	38	42	80
Regular TV	70	150	220
Total	108	192	300

- Define Event A: Customer owns an HDTV.
- What is $P(A)$?

Joint Events

- **Joint Event:** is an event that is composed of two or more events.
- Define event C as any event in either A *or* B.
  - Notation for event C: $C = A \cup B$
  - Notation for probability of event C: $P(C) = P(A \cup B)$.
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- The **complement** of an event, A, is the outcome of anything *besides* A occurring.
- Notation: $A' = A^c =$ complement of event A.
- $P(A') = 1 - P(A)$.
- Example: what is the complement of Event A: a newborn baby is a female.

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Mutually Exclusive Events

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- Are the following mutually exclusive?
  - Event A: A person is currently 8 years old. Event B: A person voted for John McCain in the last presidential election.
  - Event A: A person plays football in high school. Event B: A person plays basketball in high school.
  - Event A, Event A'.
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- Define Event A: Customer owns an HDTV.
- Define Event B: Customer owns a DVR.
- Define Event $C = A \cap B$.
- What is $P(C)$?
- Define Event $D = A \cup B$.
- What is $P(D)$?

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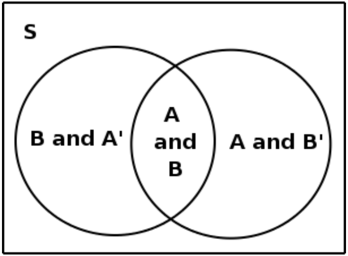

- **Venn diagram:** visualization of all possible events. The areas in the diagram represent the probabilities of those events.
- S: Event that encompasses all possible outcomes.
- Entire area of the left hand circle is $P(B)$.
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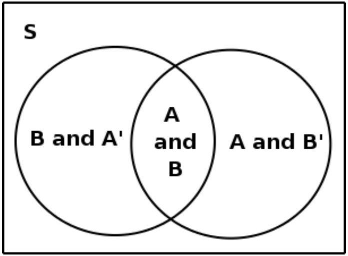

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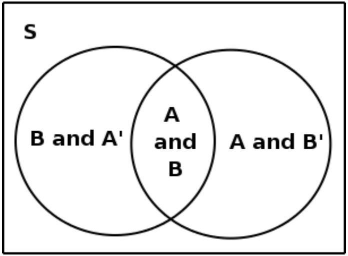

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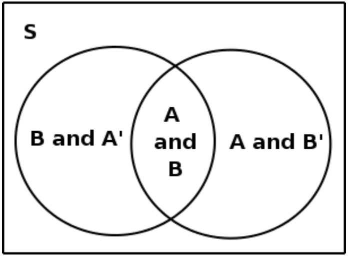

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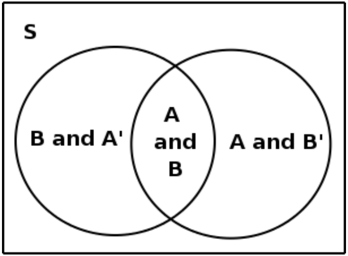

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Venn Diagram

- From the Venn Diagram we can see that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Use this equation to find the probability of owning an HDTV (Event A) or owning a DVR (Event B).

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- Example:
  - What is the probability a person is female?
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    Define event B: a person is a UW-L college student.
    $P(A|B) = 0.6$

Independence

- Two events A and B are **independent** if knowledge that A happened does not affect P(B), or if knowledge that B happened does not effect P(A).
- In the example above, is being female and being a UW-L college student independent?
- Is owning an HDTV and DVR independent?
- Is the event that someone smokes and the event someone has lung cancer independent?
- Suppose a coin is flipped twice. Is the event the first flip is heads and the event the second flip is heads independent?
- If A and B are independent, then $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

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- Why is the cool? Because this proves that:

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- Why is the cool? Because this proves that:

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Bayes Theorem

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Blood test accuracy

- Suppose a fatal disease breaks out, and a blood test is used to detect the disease.
- The blood test claims to accurately identify the disease 99% of the time.
- Let A be the event you have a disease.
- Let B be the event the blood test came out positive.

$$P(B|A) = 0.99$$

- Suppose you take the blood test and it is positive. What is the probability you have the disease?

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- Suppose 0.2% of people have the disease, and 0.198% have the disease and tested positive.

$$P(A) = 0.002$$

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Blood test accuracy

- Suppose 5% of people test positive for the disease.
- What is the probability you have the disease given you tested positive?

$$P(A|B) = \frac{P(A \cap B)}{P(A)} = \frac{0.00198}{0.05} = 0.0396$$

- Even though you tested positive, *you still most likely do not have the disease.*
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- So what is the real accuracy of things like blood tests, pregnancy tests, and lie detector tests?

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- **Random variable**: a variable that has a single numerical value determined by chance.
- Data is a bunch of realizations of a random variable.
- **Discrete random variable**: an RV that can take on "countable" values.
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Mean and variance of a probability distribution

- The mean or **expected value** of a probability distribution is:

$$\mu = \sum x_i P(x_i)$$

- The variance of a probability distribution is given by:

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Binomial distributions

- A **Bernoulli trial** results in a random variable that can only result in success ($x = 1$) or failure ($x = 0$).
  - Example: an outcome of heads for a single coin flip is a Bernoulli trial.
- A **binomial distribution** is the probability distribution for the number of successes in a fixed number of trials.
- Requirements for a binomial distribution:
  - The experiment must have a fixed number of Bernoulli trials.
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- The binomial probability distribution is given by,

$$P(x) = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x}$$

  - n: number of trials.
  - $P(x)$: the probability of x number of successes.
  - p is the probability of success for a single Bernoulli trial.
- Calculate the probability distribution of the 5 coin flip experiment.
- Verify $\sum P(x_i) = 1$.
- Calculate the expected value for the number of heads.
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Binomial Distribution

- Recall the mean of a probability distribution:

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- For the binomial distribution, this gets more simple:

$$\mu = np$$

- Recall the variance of a probability distribution:

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Normal Approximation to a Binomial

- Motivation: Questions like,
- Suppose a study concluded that Burger King fills out 10% of its orders inaccurately. If a particular franchise makes 30,000 orders a month, what is the probability it will make more than 3,100 errors in orders?
- Compute P(x=3100), P(x=3101), P(x=3102), P(x=3103), … , P(x=30,000).
- That's a ridiculous amount of computations.
- Can you suppose that the number of errors is *normally distributed* with mean equal to $\mu = np$, and variance $\sigma^2 = np(1-p)$?

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- And... the number of errors is a *discrete* random variable, but the normal distribution is for continuous random variables.
- But... the normal distribution will be a good approximation if,

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- Can you suppose that the number of errors is *normally distributed* with mean equal to $\mu = np$, and variance $\sigma^2 = np(1-p)$?
- Well... the number of errors is truly a *binomial distribution*, not a normal distribution.
- And... the number of errors is a *discrete* random variable, but the normal distribution is for continuous random variables.
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Covariance

- To measure the variance of combinations of RVs, need to know the covariance.
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Combining Random Variables

- Suppose two RVs are measured in the same units, can they be combined (added, subtracted, etc)?
- Who would want to do such a thing?
- Addition rule:
  - If $Z = X + Y$, then $E(Z) = E(X) + E(Y)$.
  - More generally, if $Z = aX + bY$, then $E(Z) = aE(X) + bE(Y)$.

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Portfolio Risk

- Don't put all your eggs in one basket.
- Would it make more sense to put your money in two investments that are negatively correlated (meaning they have a negative covariance)?
- Example, suppose the variance for the quarterly return is equal to 15% for Investment X and 10% for Investment Y, and the covariance is equal to -8%. Suppose you invested have your money in each investment.
- Would it make more sense to put your money in two investments that are positively correlated (meaning they have a positive covariance)?
- Example, suppose the variance for the quarterly return is equal to 18% for Investment X and 12% for Investment Y, and the covariance is equal to 6%. Suppose you invested have your money in each investment.

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Another Example

- Suppose you run a website business that produces and sells custom made furniture.
  - When someone submits an order online you must first build the furniture. The average time it takes to build the furniture is 9 business days and the standard deviation is 2.5 business days.
  - After you build the furniture, you must ship it. The average shipping time is 4 business days and the standard deviation is 3 business days.
  - What is the average elapsed time from the time the order is placed until the delivery?
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