Regression Analysis

BUS 735: Business Decision Making and Research

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Goals of this section

- Specific goals:
  - Learn how detect linear relationships between variables.
  - Learn how to detect relationships between ordinal and categorical variables.
  - Learn how to estimation the relationship between many variables.
- Learning objectives:
  - LO2: Be able to construct and use multiple regression models (including some limited dependent variable models) to construct and test hypotheses considering complex relationships among multiple variables.
  - LO6: Be able to use standard computer packages such as SPSS and Excel to conduct the quantitative analyses described in the learning objectives above.
  - LO7: Have a sound familiarity of various statistical and quantitative methods in order to be able to approach a business decision problem and be able to select appropriate methods to answer the question.

2 Relationships Between Two Variables

2.1 Chi-Squared Test of Independence

Chi-Squared Test for Independence

- Used to determine if two categorical variables are related.
- Example: Suppose a hotel manager surveys guest who indicate they will not return:

	Reason for Not Returning		
Reason for Stay	Price	Location	Amenities
Personal/Vacation	37	54	0
Business	34	55	19

- Data in the table are always frequencies that fall into individual categories.
- Could use this table to test if two variables are independent.

Test of independence

- **Null hypothesis**: there is no relationship between the row variable and the column variable.
- **Alternative hypothesis**: The two variables are dependent.
- Test statistic:
$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
- O: observed frequency in a cell from the contingency table.
- E: expected frequency assuming variables are independent.
- Large χ^2 values indicate variables are dependent (reject the null hypothesis).

Using SPSS

1. Open dataset *hotel.xls*.
2. Go to `Analyze`, `Descriptive Statistics`, `Crosstabs`.
3. Click `Statistics` button.
4. Check the box for `Chi-square`.
5. Click OK!

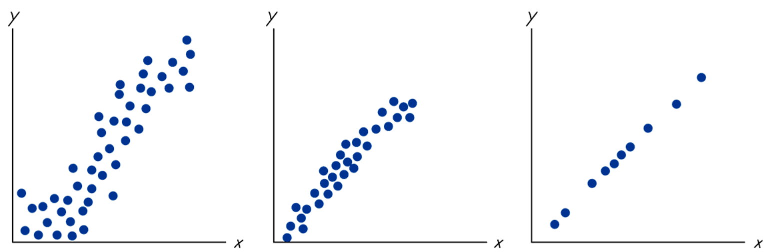
2.2 Correlation

Correlation

- A **correlation** exists between two variables when one of them is related to the other in some way.
- The **Pearson linear correlation coefficient** is a measure of the strength of the linear relationship between two variables.
  - Parametric test!
  - Null hypothesis: there is zero linear correlation between two variables.
  - Alternative hypothesis: there is [positive/negative/either] correlation between two variables.
- Spearman's Rank Test

– Non-parametric test.
– Behind the scenes - replaces actual data with their *rank*, computes the Pearson using ranks.
– Same hypotheses.

Positive linear correlation



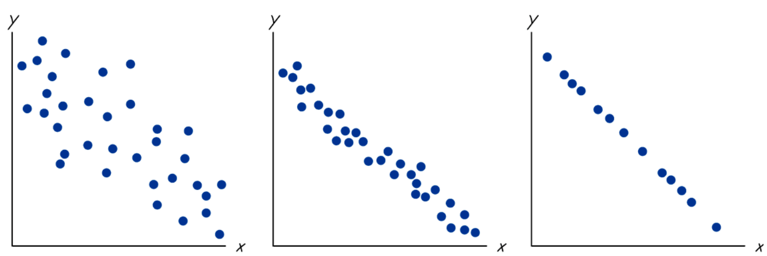
(a) Positive correlation between x and y

(b) Strong positive correlation between x and y

(c) Perfect positive correlation between x and y

- Positive correlation: two variables move in the same direction.
- Stronger the correlation: closer the correlation coefficient is to 1.
- Perfect positive correlation: $\rho = 1$

Negative linear correlation



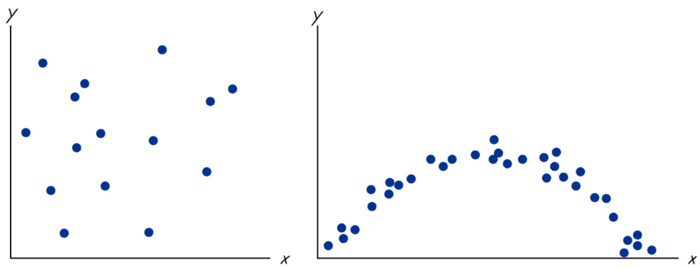
(d) Negative correlation between x and y

(e) Strong negative correlation between x and y

(f) Perfect negative correlation between x and y

- Negative correlation: two variables move in opposite directions.
- Stronger the correlation: closer the correlation coefficient is to -1.
- Perfect negative correlation: $\rho = -1$

No linear correlation



(g) No correlation between x and y

(h) Nonlinear relationship between x and y

- Panel (g): no relationship at all.
- Panel (h): strong relationship, but not a *linear* relationship.
  - Cannot use regular correlation to detect this.

Example: Public Expenditure

- Data from 1960! about public expenditures per capita, and variables that may influence it:
  - Economic Ability Index
  - Percentage of people living in metropolitan areas.
  - Percentage growth rate of population from 1950-1960.
  - Percentage of population between the ages of 5-19.
  - Percentage of population over the age of 65.
  - Dummy variable: Western state (1) or not (0).
- Is there a statistically significant linear correlation between the percentage of the population who is young and the public expenditure per capita?
- Is there a statistically significant linear correlation between the public expenditure per capita and whether or not the state is a western state?

Using SPSS

1. Open the dataset *publicexp.xls* in SPSS.
2. For a parametric test (Pearson correlation):
3. Select `Analyze` menu, select `Correlate`, then select `Bivariate`.
4. Select at least two variables (it will do all pairwise comparisons) on the left and click right arrow button.
5. Select check-box for `Pearson` and/or `Spearman`.
6. Click OK!

3 Regression

3.1 Regression Line

Regression

- Regression line: equation of the line that describes the linear relationship between variable x and variable y.
- Need to assume that *independent variables* influence *dependent variables*.
  - x: *independent* or *explanatory* variable.
  - y: *dependent* variable.
  - Variable x can influence the value for variable y, but not vice versa.
- Example: How does smoking affect lung capacity?
- Example: How does advertising affect sales?

Regression line

- Population regression line:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

- The actual coefficients β_0 and β_1 describing the relationship between x and y are unknown.
- Use sample data to come up with an estimate of the regression line:

$$y_i = b_0 + b_1 x_i + e_i$$

- Since x and y are not perfectly correlated, still need to have an error term.

Predicted values and residuals

- Given a value for x_i, can come up with a **predicted value** for y_i, denoted $\hat{y}_i$.

$$\hat{y}_i = b_0 + b_1 x_i$$

- This is not likely be the actual value for y_i.
- **Residual** is the difference *in the sample* between the actual value of y_i and the predicted value, $\hat{y}$.

$$e_i = y_i - \hat{y} = y_i - b_0 - b_1 x_i$$

Multiple Regression

- Multiple regression line (population):

$$y_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_2 + ... + \beta_{k-1} x_{k-1} + \epsilon_i$$

- Multiple regression line (sample):

$$y_i = b_0 + b_1 x_{1,i} + b_2 x_2 + ... + b_k x_k + e_i$$

  - k: number of parameters (coefficients) you are estimating.
  - ϵ_i: error term, since linear relationship between the x variables and y are not perfect.
  - e_i: residual = the difference between the predicted value $\hat{y}$ and the actual value y_i.

Least Squares Estimate

- How should we obtain the "best fitting line".
- Ordinary least squares (OLS) method.
- Choose sample estimates for the regression coefficients that minimizes:

$$\sum_{i=0}^{n} (y_i - \hat{y}_i)^2$$

Interpreting the slope

- Interpreting the slope, β: amount the y is predicted to increase when increasing x by one unit.
- When $\beta < 0$ there is a negative linear relationship.
- When $\beta > 0$ there is a positive linear relationship.
- When $\beta = 0$ there is no linear relationship between x and y.
- SPSS reports sample estimates for coefficients, along with...
  - Estimates of the standard errors.
  - T-test statistics for $H_0 : \beta = 0$.
  - P-values of the T-tests.
  - Confidence intervals for the coefficients.

Example: Public Expenditure

- Data from 1960 about public expenditures per capita, and variables that may influence it.
- In SPSS, choose `Analyze` menu and select `Regression` and `Linear`.
- Select `EX` (Expenditure per capita) as your dependent variable. This is the variable your are interested in explaining.
- Select your independent (aka explanatory) variables. These are the variables that you think can explain the dependent variable. I suggest you select these:
  - ECAB: Economic Ability
  - MET: Metropolitan
  - GROW: Growth rate of population
  - WEST: Western state = 1.

Example: Public Expenditure

- If the percentage of the population living in metropolitan areas in expected to increase by 1%, what change should we expect in public expenditure?
- Is this change statistically significantly different from zero?
- Accounting for economic ability, metropolitan population, and population growth, how much more to Western states spend on public expenditure per capita?

Using SPSS

1. Open *publicexp.xls* in SPSS.
2. Select from menu: `Analyze`, `Regression`, then `Linear`.
3. Move EX to the Dependent variable list.
4. Move ECAB, MET, GROW, and WEST to your Independent variable list.
5. Click OK!

Regression output shows:

- Coefficient of Determination (aka R^2) (more on this ahead...)
- Analysis of Variance Table (more on this ahead...)
- Coefficient Estimates, including standard errors, t-statistics, p-values

3.2 Variance Decomposition

Sum of Squares Measures of Variation

- **Sum of Squares Regression (SSR)**: measure of the amount of variability in the dependent (Y) variable that is explained by the independent variables (X's).

$$SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$$

- **Sum of Squares Error (SSE)**: measure of the unexplained variability in the dependent variable.

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Sum of Squares Measures of Variation

- **Sum of Squares Total (SST)**: measure of the total variability in the dependent variable.

$$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

- SST = SSR + SSE.

Coefficient of determination

- The **coefficient of determination** is the percentage of variability in y that is explained by x.

$$R^2 = \frac{SSR}{SST}$$

- R^2 will always be between 0 and 1. The closer R^2 is to 1, the better x is able to explain y.
- The more variables you add to the regression, the higher R^2 will be.

Adjusted R^2

- R^2 will likely increase (slightly) even by adding nonsense variables.
- Adding such variables increases in-sample fit, but will likely hurt out-of-sample forecasting accuracy.
- The Adjusted R^2 penalizes R^2 for additional variables.

$$R^2_{\text{adj}} = 1 - \frac{n-1}{n-k-1}\left(1 - R^2\right)$$

- When the adjusted R^2 increases when adding a variable, then the additional variable really did help explain the dependent variable.
- When the adjusted R^2 decreases when adding a variable, then the additional variable does not help explain the dependent variable.

F-test for Regression Fit

- F-test for Regression Fit: Tests if the regression line explains the data.
- Very, very, very similar to ANOVA F-test.
- $H_0 : \beta_1 = beta_2 = ... = \beta_k = 0$.
- H_1 : At least one of the variables has explanatory power (i.e. at least one coefficient is not equal to zero).

$$F = \frac{SSR/(k-1)}{SSE/(n-k)}$$

- Where k is the number of explanatory variables.

Example: Public Expenditure

- In the previous example, how much of the variability in public expenditure is explained by the following four variables:
  - ECAB: Economic Ability
  - MET: Metropolitan
  - GROW: Growth rate of population
  - WEST: Western state = 1.
- Is the combination of these variables significant in explaining public expenditure?
- Re-run the regression, this time also including:
  - YOUNG: Percentage of population that is young.
  - OLD: Percentage of population that is old.

Example: Public Expenditure

- What happened to the coefficient of determination?
- What happened to the adjusted coefficient of determination? What is your interpretation?
- What happened to the estimated effect of the other variables: metropolitan area? Western state?

4 Assumptions

Assumptions from the CLT

- Using the normal distribution to compute p-values depends on results from the Central Limit Theorem.
- Sufficiently large sample size (much more than 30).
  - Useful for normality result from the Central Limit Theorem
  - Also necessary as you increase the number of explanatory variables.
- Normally distributed dependent and independent variables
  - Useful for small sample sizes, but not essential as sample size increases.
- Types of data:
  - Dependent variable must be interval data or above.
  - Independent variable can be interval or above *or a dummy variable*.

Crucial Assumptions for Regression

- Linearity: a straight line reasonably describes the data.
  - Exceptions: experience on productivity, ordinal data like education level on income.
  - Consider transforming variables.
- Stationarity:
  - The central limit theorem: behavior of statistics as sample size approaches infinity!
  - The mean and variance must exist and be constant.
  - Big issue in economic and financial time series.
- Exogeneity of explanatory variables.
  - Dependent variable must not influence explanatory variables.
  - Explanatory variables must not be influenced by excluded variables that can influence dependent variable.
  - Example problem: how does advertising affect sales?

Multicollinearity

- **Multicollinearity:** when two or more of the explanatory variables are highly correlated.
- With multicollinearity, it is difficult to determine the effect coming from a specific individual variable.
- Correlated variables will have standard errors for coefficients will be large (coefficients will be statistically insignificant).
- Examples:
  - experience and age used to predict productivity
  - size of store (sq feet) and store sales used to predict demand for inventories.
  - parent's income and parent's education used to predict student performance.
- Perfect multicollinearity - when two variables are perfectly correlated.

Homoscedasticity

- **Homoscedasticity:** when the variance of the error term is constant (it does not depend on other variables).
- Counter examples (heteroscedasticity):
  - Impact of income on demand for houses.
  - Many economic and financial variables related to income suffer from this.
- Heteroscedasticity is not too problematic:
  - Estimates will still be unbiased.
  - Your standard errors will be downward biased (reject more than you should).
- May be evidence of a bigger problem: linearity or stationarity.

Using SPSS

1. Set up regression dialog as before.
2. To examine multicollinearity possibilities:
   - Check standard errors / significance levels of your coefficients - if variables that could be related are insignificant (have a large standard error), then there may be a problem.

- Compute pearson correlation coefficients for potential problematic variables.

3. To examine normality of error term:
   (a) Click `Plots`
   (b) Select checkbox for `Normal Probability Plot`.
   (c) Select checkbox for `Histogram`.
   (d) Click `Continue`
4. To examine homoscedasticity / linearity issues
   (a) Click `Save`
   (b) Under *Residuals*, select checkbox for `Standardized`.
   (c) Click `Continue`
5. Click OK.

   This will re-run the regression, create a histogram for the standardized residuals, and create a normal probability plot.
6. Select menu item `Graphs`, `Scatter/Dot`
7. Select `Simple Scatter` and click `Define`
8. Move standardized residuals to the Y-Axis, move one of the continuous explanatory variables to the X-Axis.
9. Click OK.
10. Things to look for:
    - These plots should have residuals randomly above and below zero with no discernable pattern (violation may imply a non-linear relationship).
    - Variability of residuals (how spread out they are) should not change as explanatory variable changes (violation implies heteroskedasticity).