

# Statistical Significance and Bivariate Tests

BUS 735: Business Decision Making and Research

# Goals

- Specific goals:
  - Re-familiarize ourselves with basic statistics ideas: sampling distributions, hypothesis tests, p-values.
  - Be able to distinguish different types of data and prescribe appropriate statistical methods.
  - Conduct a number of hypothesis tests using methods appropriate for questions involving only one or two variables.
- Learning objectives:
  - LO1: Be able to construct and test hypotheses using a variety of bivariate statistical methods to compare characteristics between two populations.
  - LO6: Be able to use standard computer packages such as SPSS and Excel to conduct the quantitative analyses described in the learning objectives above.

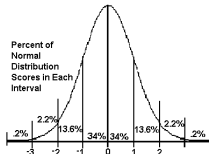
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# Probability Distribution

- **Probability distribution:** summary of all possible values a variable can take along with the probabilities in which they occur.
- Usually displayed as:

## Picture



## Table

z	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749

## Formula

$$f(x|\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

- **Normal distribution:** often used “bell shaped curve”, reveals probabilities based on how many standard deviations away an event is from the mean.

## Sampling distribution

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- Imagine taking a sample of size 100 from a population and computing some kind of statistic.
- The statistic you compute can be anything, such as: mean, median, proportion, difference between two sample means, standard deviation, variance, or anything else you might imagine.
- Suppose you repeated this experiment over and over: take a sample of 100 and compute and record the statistic.
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- Is this the same thing as the probability distribution of the population?  
**NO! They may coincidentally have the same shape though.**

## Example

- Sampling Distribution Simulator
- In reality, you only do an experiment once, so the sampling distribution is a hypothetical distribution.
- Why are we interested in this?

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- The average of the sample statistics is equal to the true population parameter.
- Want the variance of *the sampling distribution* to be as small as possible. Why?
- Want the *sampling distribution* to be normal, regardless of the distribution of the population.

# Central Limit Theorem

- Given:
  - Suppose a RV  $x$  has a distribution (it need not be normal) with mean  $\mu$  and standard deviation  $\sigma$ .
  - Suppose a *sample mean* ( $\bar{x}$ ) is computed from a sample of size  $n$ .
- Then, if  $n$  is sufficiently large, the sampling distribution of  $\bar{x}$  will have the following properties:
  - The sampling distribution of  $\bar{x}$  will be normal.
  - The mean of the sampling distribution will equal the mean of the population (consistent):

$$\mu_{\bar{x}} = \mu$$

- The standard deviation of the sampling distribution will decrease with larger sample sizes, and is given by:

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Suppose average birth weight is  $\mu = 7\text{lbs}$ , and the standard deviation is  $\sigma = 1.5\text{lbs}$ .

What is the probability that a sample of size  $n = 30$  will have a mean of  $7.5\text{lbs}$  or greater?

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The probability the sample mean is greater than  $7.5\text{lbs}$  is:

$$P(\bar{x} > 7.5) = P(z > 1.826) = 0.0339$$



## Example 2

Suppose average birth weight is  $\mu = 7\text{lbs}$ , and the standard deviation is  $\sigma = 1.5\text{lbs}$ .

What is the probability that a randomly selected baby will have a weight of 7.5lbs or more? What do you need to assume to answer this question?

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The probability that a baby is greater than 7.5lbs is:

$$P(x > 7.5) = P(z > 0.33) = 0.3707$$

## Example 3

- Suppose average birth weight of all babies is  $\mu = 7lbs$ , and the standard deviation is  $\sigma = 1.5lbs$ .
- Suppose you collect a sample of 30 newborn babies whose mothers used illegal drugs during pregnancy.
- Suppose you obtained a sample mean  $\bar{x} = 6lbs$ . If you assume the mean birth weight of babies whose mothers used illegal drugs has the same sampling distribution as the rest of the population, what is the probability of getting a sample mean this low or lower?

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That is, if using drugs during pregnancy actually does still lead to an average birth weight of 7 pounds, there was only a 0.000131 (or 0.0131%) chance of getting a sample mean as low as six or lower. This is an extremely unlikely event if the assumption is true. Therefore it is likely the assumption is not true.

# Statistical Hypotheses

- A **hypothesis** is a claim or statement about a property of a population.
  - Example: The population mean for systolic blood pressure is 120.
- A **hypothesis test** (or **test of significance**) is a standard procedure for testing a claim about a property of a population.
- Recall the example about birth weights with mothers who use drugs.
  - Hypothesis: Using drugs during pregnancy leads to an average birth weight of 7 pounds (the same as with mothers who do not use drugs).



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# Null and Alternative Hypotheses

- The **null hypothesis** is a statement that the value of a population parameter (such as the population mean) *is equal* to some claimed value.
  - $H_0: \mu = 7.$
- The **alternative hypothesis** is an alternative to the null hypothesis; a statement that says a parameter differs from the value given in the null hypothesis.
  - $H_a: \mu < 7.$
  - $H_a: \mu > 7.$
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- In hypothesis testing, assume the null hypothesis is true until there is strong statistical evidence to suggest the alternative hypothesis.
- Similar to an “innocent until proven guilty” policy.

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  - $H_a: \mu > 7.$
  - $H_a: \mu \neq 7.$
- In hypothesis testing, assume the null hypothesis is true until there is strong statistical evidence to suggest the alternative hypothesis.
- Similar to an “innocent until proven guilty” policy.

# Hypothesis tests

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- (Many) hypothesis tests are all the same:

$$z \text{ or } t = \frac{\text{sample statistic} - \text{null hypothesis value}}{\text{standard deviation of the sampling distribution}}$$

- Example: hypothesis testing about  $\mu$ :
  - Sample statistic =  $\bar{x}$ .
  - Standard deviation of the sampling distribution of  $\bar{x}$ :

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## P-values

- The P-value of the test statistic, is the area of the sampling distribution from the sample result in the direction of the alternative hypothesis.
- Interpretation: If the null hypothesis is correct, than the p-value is the probability of obtaining a sample that yielded your statistic, or a statistic that provides even stronger evidence of the null hypothesis.
- The p-value is therefore a measure of *statistical significance*.
  - If p-values are very small, there is strong statistical evidence in favor of the alternative hypothesis.
  - If p-values are large, there is insignificant statistical evidence. When large, you fail to reject the null hypothesis.
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## Types of Data

- Nominal data: consists of categories that cannot be ordered in a meaningful way.
- Ordinal data: order is meaningful, but not the distances between data values.
  - Excellent, Very good, Good, Poor, Very poor.
- Interval data: order is meaningful, *and* distances are meaningful. However, there is *no natural zero*.
  - Examples: temperature, time.
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# Single Mean T-Test

- Test whether the population mean is equal or different to some value.
- Uses the sample mean its statistic.
- Why T-test instead of Z-test?
- Parametric test that depends on results from Central Limit Theorem.
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  - Null: The population mean is equal to some specified value.
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## Example: Public School Spending

- Dataset: average pay for public school teachers and average public school spending per pupil for each state and the District of Columbia in 1985.
- Download dataset `eduspending.xls`.
- Conduct the following exercises:
  - Show some descriptive statistics for teacher pay and expenditure per pupil.
  - Is there statistical evidence that teachers make less than \$25,000 per year?
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## Example: Economic Outlook

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- Data from Montana residents in 1992 concerning their outlook for the economy.
- All data is ordinal or nominal:
  - AGE = 1 under 35, 2 35-54, 3 55 and over
  - SEX = 0 male, 1 female
  - INC = yearly income: 1 under \$20K, 2 20-35\$K, 3 over \$35K
  - POL = 1 Democrat, 2 Independent, 3 Republican
  - AREA = 1 Western, 2 Northeastern, 3 Southeastern Montana
  - FIN = Financial status 1 worse, 2 same, 3 better than a year ago
  - STAT = 0, State economic outlook better, 1 not better than a year ago
- Do the majority of Montana residents feel their financial status is the same or better than one year ago?
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# Single Median Nonparametric Test

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- Why?
  - Ordinal data: cannot compute sample means (they are meaningless), only median is meaningful.
  - Small sample size and you are not sure the population is not normal.
- Sign test: can use tests for proportions for testing the median.
  - For a null hypothesized population median...
  - Count how many observations are above the median.
  - Test whether that proportion is greater, less than, or not equal to 0.5.
  - For small sample sizes, use binomial distribution instead of normal distribution.

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- Open dataset `gradschools.xls`. Choose second worksheet, titled Data.
- Answer some of these questions:
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# Correlation

- A **correlation** exists between two variables when one of them is related to the other in some way.
- The **Pearson linear correlation coefficient** is a measure of the strength of the linear relationship between two variables.
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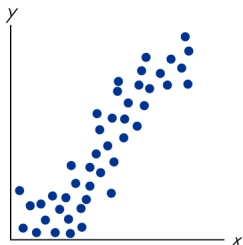


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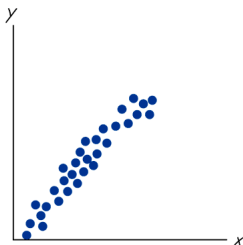
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## Positive linear correlation

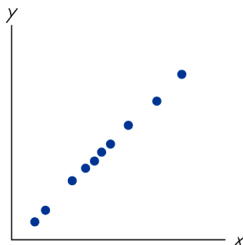
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(a) Positive correlation between  $x$  and  $y$



(b) Strong positive correlation between  $x$  and  $y$

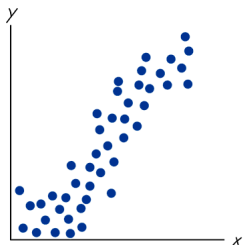


(c) Perfect positive correlation between  $x$  and  $y$

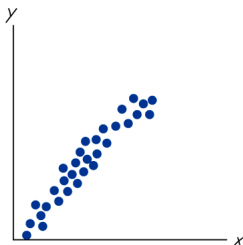
- Positive correlation: two variables move in the same direction.
- Stronger the correlation: closer the correlation coefficient is to 1.
- Perfect positive correlation:  $\rho = 1$

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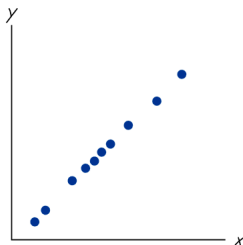
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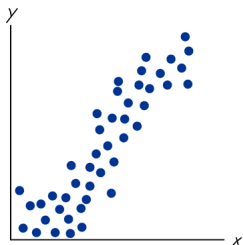


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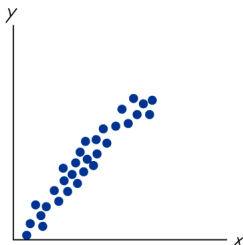
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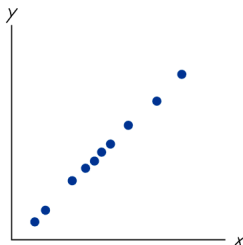
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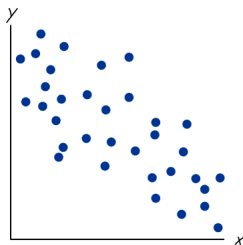


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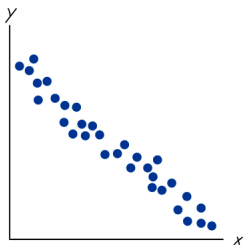
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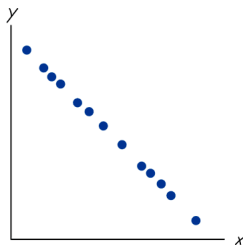
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(d) Negative correlation between  $x$  and  $y$



(e) Strong negative correlation between  $x$  and  $y$

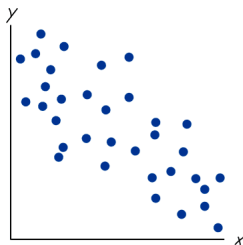


(f) Perfect negative correlation between  $x$  and  $y$

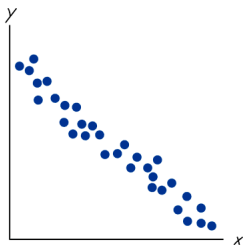
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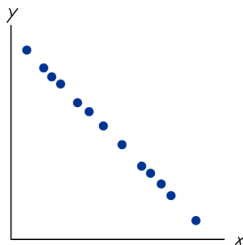
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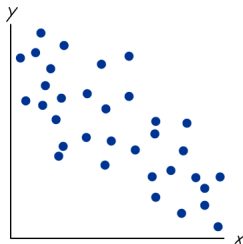
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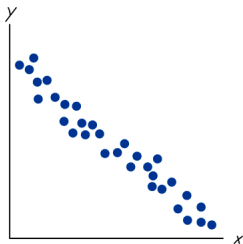
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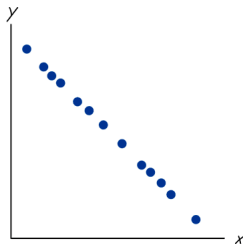
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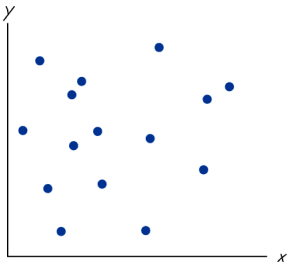


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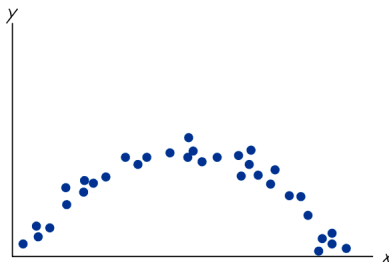
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## No linear correlation

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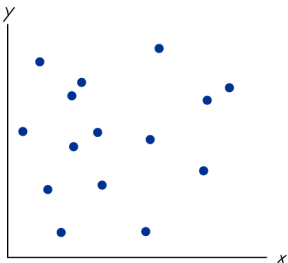
(h) Nonlinear relationship  
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- Panel (g): no relationship at all.
- Panel (h): strong relationship, but not a *linear* relationship.
  - Cannot use regular correlation to detect this.

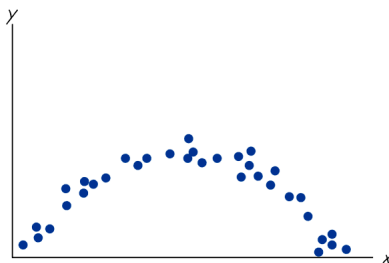


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## Example: Public Expenditure

- Data from 1960! about public expenditures per capita, and variables that may influence it:
  - Economic Ability Index
  - Percentage of people living in metropolitan areas.
  - Percentage growth rate of population from 1950-1960.
  - Percentage of population between the ages of 5-19.
  - Percentage of population over the age of 65.
  - Dummy variable: Western state (1) or not (0).
- Is there a statistically significant linear correlation between the percentage of the population who is young and the public expenditure per capita?
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## Difference in Means (Independent Samples)

29 / 34

- Suppose you want to know whether the mean from one population is larger than the mean for another.
- Examples:
  - Compare sales volume for stores that advertise versus those that do not.
  - Compare production volume for employees that have completed some type of training versus those who have not.
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# Independent Samples T-Test

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- Hypotheses:
  - Null hypothesis: the difference between the two means is zero.
  - Alternative hypothesis: the difference is [above/below/not equal] to zero.
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  - To guide you, SPSS also reports Levene's test for equality of variance (Null - variances are the same).

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# Nonparametric Tests for Differences in Medians

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- Mann-Whitney U test: nonparametric test to determine difference in *medians*.
- Assumptions:
  - Samples are independent of one another.
  - The underlying distributions have the same shape (i.e. only the location of the distribution is different).
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## Dependent Samples - Paired Samples

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- Use a **paired sampled test** if instead the two samples have the same individuals before and after some treatment.
- Really simple: for each individual subtract the before treatment measure from the after treatment measure (or vice-versa).
- Treat your new series as a single series.
- Conduct one-sample tests.
- In SPSS, you need to have separate columns for each of these variables.
- There are methods in SPSS specifically for Dependent Samples tests - but the paired sampled approaches are equivalent to one-sample tests.

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## Dependent Samples - Paired Samples

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  - When it is appropriate to use parametric versus non-parametric methods.
  - Most univariate and bivariate questions have a parametric and non-parametric approach.
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  - Chapter 1 end of chapter exercises (these may include hypothesis tests we did not cover!), Part I and II.
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