

## Overview of Statistical Methods / ANOVA

BUS 735: Business Decision Making and Research

# Goals

- Specific goals:
  - Re-familiarize ourselves with statistical tests.
  - Learn how to choose appropriate tests.
  - Learn how to compare means or medians among more than two populations.
- Learning objectives:
  - LO1: Be able to construct and test hypotheses using a variety of bivariate statistical methods to compare characteristics between two populations.
  - LO3: Be able to construct and use analysis of variance and analysis of covariance models to construct and test hypotheses considering complex relationships among multiple variables.
  - LO6: Be able to use standard computer packages such as SPSS and Excel to conduct the quantitative analyses described in the learning objectives above.

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## Selecting Right Method

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- Parametric Methods:
  - Only for *interval or ratio data*.
  - Make sure assumptions of CLT hold:
    - Large sample size *or..*
    - Normal distributed *population*.
- Non-parametric methods using ranks
  - Ordinal data *and/or...*
  - Central limit theorem does not apply.
- Non-parametric Chi-squared test
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# Single Population

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- Examine a proportion
  - Parametric: treat data as 0s and 1s, T-test for a single mean.
  - Nonparametric: Binomial distribution.
- Examine the “average” (measure of center) of a single population.
  - Parametric method: T-test for a single mean.
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# Differences in Two Populations

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- Independent Samples
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- Paired samples (Dependent Samples)
  - Parametric: Paired samples T-Test
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## Relationships Between Two Variables

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- Nonparametric method: Spearman correlation.
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# One-Way ANOVA

- Method for testing for significant differences among means from two or more groups.
- Essentially an extension of the t-test for testing the differences between two means.
- Uses measures of *variance* to measure for differences in *means*.
- Total variation in your data is decomposed into two components:
  - **Among-group variation:** variability that is due to differences among groups, also called *explained* variation.
  - **Within-group variation:** total variability within each of the groups, this is unexplained variation.

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# Hypothesis Test

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- Null hypothesis:  $\mu_1 = \mu_2 = \dots = \mu_K$
- Alternative hypothesis: At least one of the means are different from the others.
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## Assumptions behind One-way ANOVA F-test

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- Randomness: individual observations are assigned to groups *randomly*.
- Independence: individuals in each group are independent from individuals in another group.
- Sufficiently large (?) sample size, or else population must have a normal distribution.
- Homogeneity of variance: the variances of each of the  $K$  groups must be equal ( $\sigma_1^2 = \sigma_2^2 = \dots \sigma_K^2$ ).
  - Levene test for homogeneity of variance can be used to test for this.

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## Example: Crime Rates

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- Data on 47 states from 1960 (I know its old) on the crime rate and a number of factors that may influence the crime rate.
- In particular, I made a variable that put unemployment into categories:
  - Unemployment = 1 if unemployment rate was less than 8%.
  - Unemployment = 2 if unemployment rate was between 8 and 10%.
  - Unemployment = 3 if unemployment rate was greater than 10%.
- I also made a variable that categorized schooling:
  - Schooling = 1 if mean years of schooling for given state was less than 10 years.
  - Schooling = 2 otherwise.
- Is there statistical evidence that the mean crime rate is different among the different categories for the level of unemployment?

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## FYI: Explanation of all the variables

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- R: Crime rate: # of offenses reported to police per million population
- Age: The number of males of age 14-24 per 1000 population
- S: Indicator variable for Southern states (0 = No, 1 = Yes)
- Ed: Mean # of years of schooling  $\times$  10 for persons of age 25 or older
- Ex0: 1960 per capita expenditure on police by state and local government
- Ex1: 1959 per capita expenditure on police by state and local government
- LF: Labor force participation rate per 1000 civilian urban males age 14-24
- M: The number of males per 1000 females
- N: State population size in hundred thousands
- NW: The number of non-whites per 1000 population
- U1: Unemployment rate of urban males per 1000 of age 14-24
- U2: Unemployment rate of urban males per 1000 of age 35-39
- W: Median value of transferable goods and assets or family income in tens of \$
- X: The number of families per 1000 earning below  $1/2$  the median income

## Using SPSS to Conduct One-way ANOVA Tests

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- 1 Download and open the dataset `crime.sav` in SPSS.
- 2 Click on Analyze menu, then Compare Means, then select One-Way ANOVA.
- 3 Move Crime rate to the Dependent List.
- 4 Move Unemployment to Factor.
- 5 For extra tests:
  - Click on Post-hoc button for tests to compare pair-wise differences in the means.
  - Click on Options button for descriptive statistics for for homogeneity of variance test.

## One-way ANOVA output

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- 1 Descriptive Statistics: shows the mean unemployment rate for each of the three groups, also includes standard deviation, standard error, and confidence intervals. It's nice to present such statistics in your papers.
- 2 Levene's Test of Homogeneity of Variances. The null hypothesis is that the variances are equal.
- 3 ANOVA Table: presents the sum of squares, the mean sum of squares, the F-statistic, and the p-value.
- 4 Tukey Tests for all pairwise comparisons.

# Nonparametric One-way ANOVA

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- Kruskal-Wallis Rank Test: non-parametric technique for testing for differences in the *medians* among two or more groups.
- Like the Mann-Whitney U-test, uses information about the ranks of the observations, instead of the actual sizes.
- Null hypothesis:  $\theta_1 = \theta_2 = \dots = \theta_K$  (i.e. all groups have the same median).
- Alternative hypothesis: at least one of the medians differ.
- As the sample size gets large (over 5 per group some say!), the Kruskal-Wallis test statistic approaches a  $\chi^2$  distribution with  $K - 1$  degrees of freedom.
- For small sample sizes: possible to compute exact p-values without depending on asymptotic distributions.

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## Assumptions for Kruskal-Wallis Test

- Randomness: individual observations are assigned to groups *randomly*.
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- Only the location (i.e. the center) of the distributions differ among the groups. The populations otherwise have the same distribution.

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## Using SPSS to Conduct Kruskal-Wallis Test

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- 1 Click on Analyze menu, then Nonparametric Tests, then select K-Independent Samples.
- 2 Move Crime rate to Test Variable List.
- 3 Move Unemployment to Grouping Variable.
- 4 Make sure Kruskal-Wallis H text box is selected.
- 5 Click on Exact button if you need exact p-values.
- 6 Click OK!
- 7 Results show average ranks for each group and  $\chi^2$  test statistic and p-values.