Introduction to Probability

BUS 735: Business Decision Methods and Research

Goals and Agenda

Learning Objective	Active Learning Activity
Learn basics of probability.	Lecture / Practice Problems
Learn what are probability distributions	Lecture / Practice Problems.
Learn specific probability distributions: Binomial Distribution, Normal Distribution	Lecture / Practice problems.
Learn how to combine random variables.	Lecture / Practice Problems.
Practice what we have learned.	Group Exercise.
More practice.	Read Chapter 11, Homework exercises.
Assess what we have learned	Quiz

Basic Probability

- **Probability:** numeric value between 0 and 1 (or 0% and 100%) representing the chance, likelihood, or possibility some event will occur.
- **Event:** some possible (or even impossible) outcome occurring.
  - Denote events with capital English letters.
- Example:
  - A: A newborn baby will be female.
  - $P(A) = 0.5$ means there is a 50% chance that a newborn baby is female.
- Computing probability:

$$P(A) = \frac{n(A)}{T}$$

  - $n(A)$ = number of ways event A can occur.
  - T = total number of possible outcomes.

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Contingency Table

	Purchased DVR		
TV Purchased	Yes	No	Total
HDTV	38	42	80
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- Define Event A: Purchased an HDTV.
- What is $P(A)$?

Joint Events

- **Joint Event:** is an event that is composed of two or more events.
- Define event C as any event in either A *or* B.
  - Notation for event C: $C = A \cup B$
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Complements of Events

- The **complement** of an event, A, is the outcome of anything *besides* A occurring.
- Notation: $A' = A^c =$ complement of event A.
- $P(A') = 1 - P(A)$.
- Example: what is the complement of Event A: a newborn baby is a female.

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Mutually Exclusive Events

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  - Event A: A person plays football in high school. Event B: A person plays basketball in high school.
  - Event A, Event A'.
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- Define Event A: Purchased an HDTV.
- Define Event B: Purchased a DVR.
- Define Event $C = A \cap B$.
- What is $P(C)$?
- Define Event $D = A \cup B$.
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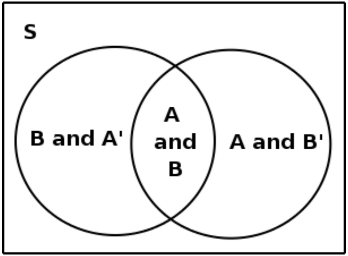

- **Venn diagram:** visualization of all possible events. The areas in the diagram represent the probabilities of those events.
- S: Event that encompasses all possible outcomes.
- Entire area of the left hand circle is $P(B)$.
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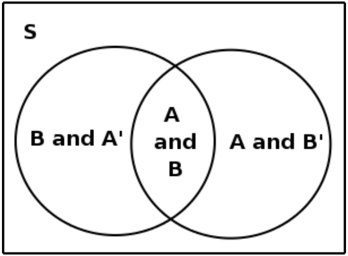

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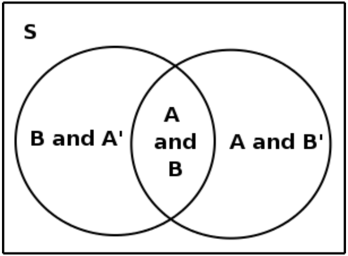

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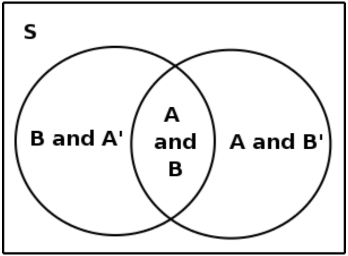

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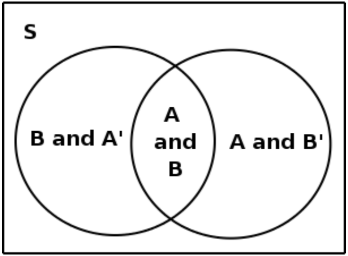

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Venn Diagram

- From the Venn Diagram we can see that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Use this equation to find the probability of owning an HDTV (Event A) or owning a DVR (Event B).

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    Define event B: a person in a nurse.
    $P(A|B) = 0.8$ (I just made that up)

Independence

- Two events A and B are **independent** if knowledge that A happened does not affect the probability that B occurs, or if knowledge that B happened does not effect the probability that A occurs.
- In the example above, is being female and being a nurse independent?
- More examples:
  - Is the event that someone smokes and the event someone has lung cancer independent?
  - Suppose a coin is flipped twice. Is the event the first flip is heads and the event the second flip is heads independent?
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$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Why is the cool? Because this proves that:

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- Why is the cool? Because this proves that:

$$P(A|B) \neq P(B|A)$$

Bayes Theorem

- Not so cool example: Suppose $P(A) = 0.4$, $P(B) = 0.8$, and $P(A \cap B) = 0.2$. What is $P(A|B)$?

$$P(A|B) = \frac{0.2}{0.8} = 0.25$$

- Are events A and B independent?

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Blood test accuracy

- Suppose a fatal disease breaks out, and a blood test is used to detect the disease.
- The blood test claims to accurately identify the disease 99% of the time.
- Let A be the event you have a disease.
- Let B be the event the blood test came out positive.

$$P(B|A) = 0.99$$

- Suppose you take the blood test and it is positive. What is the probability you have the disease?

$$P(A|B) = ?$$

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- Suppose 0.2% of people have the disease, and 0.198% have the disease and tested positive.

$$P(A) = 0.002$$

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Blood test accuracy

- Suppose 5% of people test positive for the disease.
- What is the probability you have the disease given you tested positive?

$$P(A|B) = \frac{P(A \cap B)}{P(A)} = \frac{0.00198}{0.05} = 0.0396$$

- Even though you tested positive, *you still most likely do not have the disease.*
- And the test had the claim of being 99% accurate.

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Test Accuracies?

So what is the real accuracy of things like blood tests, pregnancy tests, and lie detector tests?

Definitions

- **Random variable**: a variable that has a single numerical value determined by chance.
- Data is a bunch of realizations of a random variable.
- **Discrete random variable**: an RV that can take on "countable" values.
- **Continuous random variable**: a RV that can take on infinitely many values on a continuous scale.

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Probability distribution

- A **probability distribution** is a graph, table, or formula that gives the probability for each value of a random variable.
- The sum of the probabilities for all possible values an RV can take must equal 1 (or 100%).

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Mean and variance of a probability distribution

- The mean or **expected value** of a probability distribution is:

$$\mu = \sum x_i P(x_i)$$

- The variance of a probability distribution is given by:

$$\sigma^2 = \sum \left[(x_i - \mu)^2 P(x_i) \right]$$

$$\sigma^2 = \sum \left(x_i^2 P(x_i) \right) - \mu^2$$

- Try calculating the mean, variance, and standard deviation for the previous example.

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Binomial distributions

- A **Bernoulli trial** results in a random variable that can only result in success ($x = 1$) or failure ($x = 0$).
  - Example: an outcome of heads for a single coin flip is a Bernoulli trial.
- A **binomial distribution** is the probability distribution for the number of successes in a fixed number of trials.
- Requirements for a binomial distribution:
  - The experiment must have a fixed number of Bernoulli trials.
  - The trials must be independent.
  - The probability of success must be the same for each trial.
- Example: What are the possible outcomes for the number of successes in 3 coin flips?

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- The binomial probability distribution is given by,

$$P(x) = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x}$$

  - n: number of trials.
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- Calculate the probability distribution of the 3 coin flip experiment.
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Binomial Distribution

- Recall the mean of a probability distribution:

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- For the binomial distribution, this gets more simple:

$$\mu = np$$

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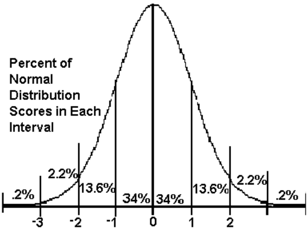
Normal Distribution

- A very specific symmetric "bell shaped" curve that predicts precise probabilities for ranges of values.
- Probabilities depend on how far an observation is away from the mean.



- Horizontal Axis: number of standard deviations away from the mean.
- Area under the curve represents probability.
- Formula:
$$f(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

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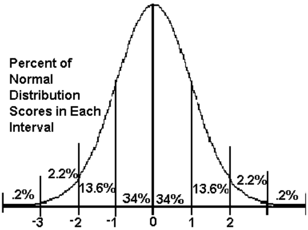

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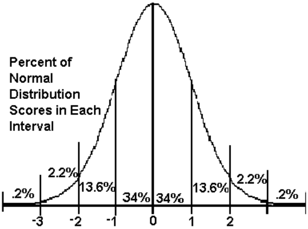

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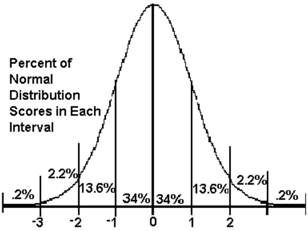

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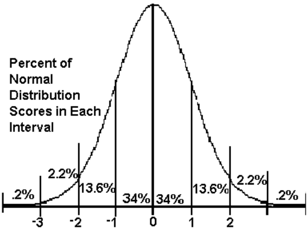
- A very specific symmetric “bell shaped” curve that predicts precise probabilities for ranges of values.
- Probabilities depend on how far an observation is away from the mean.

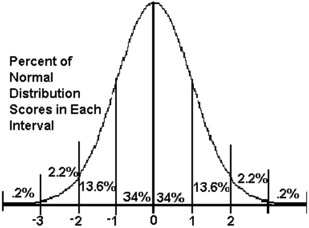


- Horizontal Axis: number of standard deviations away from the mean.
- Area under the curve represents probability.
- Formula:

$$f(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Normal Distribution

German mathemetician and scientist Carl Friedrich Gauss (1777-1855) derived the normal distribution.



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Duetsche Mark

Computing Normal Probabilities Using Excel

- `=normsdist('val')` returns the $P(z <' val')$.
- Examples: Suppose Shep's Shoe Shop November sales revenue is normally distributed and has a mean of $3,500 with a standard deviation of $800.
  - Suppose Shep's monthly fixed costs are $2,000. What is the probability November sales fail to cover fixed costs?
  - Shep only has enough inventory to sell $4,500 worth of shoes. What is the probability his sales will exceed his inventory?

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Normal Approximation to a Binomial

- Motivation: Questions like,
- Suppose a study concluded that Burger King fills out 10% of its orders inaccurately. If a particular franchise makes 30,000 orders a month, what is the probability it will make more than 3,100 errors in orders?
- Compute P(x=3100), P(x=3101), P(x=3102), P(x=3103), ... , P(x=30,000).
- That's a ridiculous amount of computations.
- Can you suppose that the number of errors is *normally distributed* with mean equal to $\mu = np$, and variance $\sigma^2 = np(1-p)$?

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- Can you suppose that the number of errors is *normally distributed* with mean equal to $\mu = np$, and variance $\sigma^2 = np(1-p)$?
- Well... the number of errors is truly a *binomial distribution*, not a normal distribution.
- And... the number of errors is a *discrete* random variable, but the normal distribution is for continuous random variables.
- But... the normal distribution will be a good approximation if,

$$np > 5 \text{ and } n(1-p) > 5$$

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Covariance

- To measure the variance of combinations of RVs, need to know the covariance.
- **Covariance**: measure of how two RVs move together.
- Notation:
  - σ_{xy}: population covariance.
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- Interpretations:
  - When covariance is negative, variables move in opposite directions.
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Combining Random Variables

- Suppose two RVs are measured in the same units, can they be combined (added, subtracted, etc)?
- Who would want to do such a thing?
- Addition rule:
  - If $Z = X + Y$, then $E(Z) = E(X) + E(Y)$.
  - More generally, if $Z = aX + bY$, then $E(Z) = aE(X) + bE(Y)$.

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$$VAR(X+Y) = VAR(X) + VAR(Y) + 2COV(X,Y).$$

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Portfolio Risk

- Don't put all your eggs in one basket.
- Would it make more sense to put your money in two investments that are negatively correlated (meaning they have a negative covariance)?
- Example, suppose the variance for the quarterly return is equal to 15% for Investment X and 10% for Investment Y, and the covariance is equal to -8%. Suppose you invested have your money in each investment.
- Would it make more sense to put your money in two investments that are positively correlated (meaning they have a positive covariance)?
- Example, suppose the variance for the quarterly return is equal to 18% for Investment X and 12% for Investment Y, and the covariance is equal to 6%. Suppose you invested have your money in each investment.

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Next time...

- (Re)read the textbook on this topic (BWT, Chapter 11).
- Homework assignment: End of Chapter 11 problems 7, 9, 11c, 13c, 19, 21, 31, 33, 37.
- Quiz on this topic.
- Next topic: Decision Analysis (BWT, Chapter 12).