

# Transportation Models

BUS 735: Business Decision Making and Research

| <b>Learning Objective</b>  | <b>Active Learning Activity</b> |
|--|---------------------------------|
| Learn what transportation models are.  | Lecture / Discussion            |
| Learn how to formulate transportation problems.  | Worksheet problem.              |
| Build our expertise and experience with transportation models  | Quiz                            |
| Learn how transportation models can be useful for problems not really involving transporting things. |                                 |
| Learn what assignment models are.  | Lecture / Discussion            |
| Learn how handle assignments with a transportation model.  | Worksheet problem.              |

## Goals of this class meeting

- Specific Goals:
  - Learn how to formulate models involving transporting goods from suppliers to destinations.
  - Learn how to use the transportation model framework for finding optimal assignments.
  - Continue to perfect our linear programming / Excel skills!
- Learning Objectives:
  - Be able to construct and solve linear programming models to answer business optimization problems.
  - Be able to use standard computer packages such as SPSS and Excel to conduct the quantitative analyses described in the learning objectives above.

# 1 Transportation Models

## Transportation Models

- **Transportation Models:** class of problems involving transporting goods from suppliers to destinations, usually at minimum cost.
- Assumptions:
  - Each source has a fixed supply (not essential).
  - Each destination has a fixed demand (not essential).
- The cost of transporting goods differs between points.
- Meeting demand at each location typically requires supply from multiple sources.

## Example: Getting grain to mills

- We have three different grain elevators scattered around the Great Plains that can supply grain:
  1. Kansas City supplies 150 tons of grain.
  2. Omaha supplies 175 tons of grain.
  3. Des Moines supplies 275 tons of grain.
- We have three different grain mills that need grain:
  1. Chicago needs 200 tons of grain.
  2. St. Louis needs 100 tons of grain.
  3. Cincinnati needs 300 tons of grain.

### Transportation Costs

Differing distances between locations, and different gasoline prices along the routes, lead to different costs for transportation.

| Source Cities   | Destination Cities |               |                |
|-----------------|--------------------|---------------|----------------|
|                 | (A) Chicago        | (B) St. Louis | (C) Cincinnati |
| (1) Kansas City | \$6                | \$8           | \$10           |
| (2) Omaha       | \$7                | \$11          | \$11           |
| (3) Des Moines  | \$4                | \$5           | \$12           |

### What do we do?

- We want to know how much grain we should send from each grain elevator to each grain mill.
- What is our objective?
- What are our choice variables? How many are there?
- Notation: let  $x_{1A}$  denote the amount of grain coming from source 1 (Kansas City) to destination A (Chicago).

### Worksheet questions

1. Write the objective function.
  
  
  
  
  
  
  
  
  
  
2. There are two sources of constraints, what are they?

3. How does the total supply compare with the total demand? This will determine whether you should use equality or inequality constraints.

4. Write down all the constraints.

5. Use Excel Solver to find the solution. What is the minimum cost?

### Model Balance

- A **balanced transportation model** is one where total demand is equal to total supply.
  - All supplies will be used  $\rightarrow$  supply constraints have  $=$ .
  - All demands are satisfied  $\rightarrow$  demand constraints have  $=$ .
- If demand exceeds supply (**unbalanced transportation model**):
  - All supplies will be used  $\rightarrow$  supply constraints have  $=$ .
  - Not all demands can be satisfied  $\rightarrow$  demand constraints have  $\leq$ .
- If supply exceeds demand (**unbalanced transportation model**):
  - Not all supplies will be used  $\rightarrow$  supply constraints have  $\leq$ .
  - All demands can be satisfied  $\rightarrow$  demand constraints have  $=$ .

## Quiz!

A large manufacturing company is downsizing amid tough economic times and seemingly permanent changes in the profitability of the manufacturing sector in the United States. The company is closing three plants: Pittsburgh, PA; Gary, IN; and Detroit, MI. There are some highly skilled employees working at these plants they would like to keep and send to their overseas plants in Bangladesh, Thailand, and India.

The number of skilled employees who have agreed to transfer overseas at each plant are as follows:

| <b>Closing Plant</b> | <b>Transferable Employees</b> |
|----------------------|-------------------------------|
| Pittsburgh, PA       | 60                            |
| Gary, IN             | 105                           |
| Detroit, MI          | 70                            |

The number of employees that can be accommodated at each of the overseas plants are as follows:

| <b>Oversees Plant</b> | <b>Employees Demanded</b> |
|-----------------------|---------------------------|
| Bangladesh            | 45                        |
| Thailand              | 90                        |
| India                 | 35                        |

Due to differences in the type of manufactured goods produced at each plant and the differences in the manufacturing processes, not all transferable employees will be equally productive at the various overseas operations. The increase in profitability per year at the overseas plants that results from transferring a highly trained American employee depends on what plant she/he is coming from and what plant she/he is going to. The following table shows how much annual profit will increase by for each employee depending on their experience and their new assignment:

| <b>Employee's Original Plant</b> | <b>Oversees Plant Assignment</b> |          |        |
|----------------------------------|----------------------------------|----------|--------|
|                                  | Bangladesh                       | Thailand | India  |
| Pittsburgh, PA                   | \$50K                            | \$80K    | \$60K  |
| Gary, IN                         | \$100K                           | \$90K    | \$120K |
| Detroit, MI                      | \$70K                            | \$60K    | \$80K  |

The company wants to transfer the employees so as to maximize profits.

## Quiz!

1. What are your decision variables? How many do you have? Carefully define the notation you decide to use.
2. What is your objective function?
3. There are two sources for constraints. Describe them. Do you have a balanced or an unbalanced model?

4. Write down all of your constraints.

5. Solve the problem in Excel. Print out the results. Point out how many employees are transferred between plants, and what the maximum profit is for the manufacturing company.

## 2 Assignment Models

### Assignment Models

- **Assignment models** are like transportation models, except you decide whether or to assign a source to a destination (or employee to a task).
- Decision variables are **binary**.
- Suppose you have 3 employees and 3 tasks. How many different possible assignments are there?
- Constraints:
  - Each assignment must get at most 1 assignee.
  - Each assignee must get at most 1 assignment.
  - Non-negativity constraints.
  - Integer constraints (use Integer Programming).

### Worksheet questions

Fancy Pants Department Store has four salespeople with a great deal of experience throughout the store, and is ready to give three promotions to department supervisor. The three departments that need supervisors are Women's clothing, Men's Clothing, and Jewelry. Each employee has worked previously in all these departments, and the store keeps data on sales for each employee for each department. The employees' average monthly sales numbers are,

| Salesperson | Department |       |         |
|-------------|------------|-------|---------|
|             | Men        | Women | Jewelry |
| Abby        | \$340      | \$160 | \$610   |
| Beth        | \$560      | \$370 | \$520   |
| Carla       | \$270      | \$540 | \$350   |
| Deborah     | \$360      | \$220 | \$630   |

The store manager thinks it is reasonable to promote the employees to supervise departments in such a way that maximizes the expected sales of the department. What assignments should be made to achieve the managers objective? Does everyone get promoted? What are the total expected sales for the supervisors?

### Homework

- Transportation models: Pages 251-252, problems 15, 16, and 17.