Network Models

BUS 735: Business Decision Making and Research

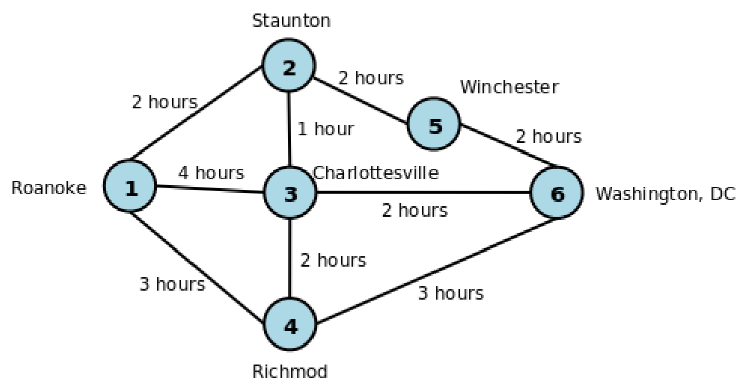
Learning Objective	**Active Learning Activity**
Learn how to solve minimum distance network problems	Lecture Example solved by hand.
Learn how to solve minimal spanning tree problems	Lecture Example solved by hand.
Learn how to solve maximal flow problems	Lecture Example solved with a computer.
Assess what we have learned.	Quiz

Network Flow Model: Shortest Route

- In these problems, there is only one item (person/truck/etc) that comes from one source and arrives one destination.
- The problem: there are lots of intermediate points (**nodes**) and lots of routes to choose from.
- **Branch**: path from one node to an adjacent node.

Example

- Goal: Drive from Roanoke, VA to Washington DC, using the fastest route (without using Google Maps).
- The possible routes and distances are given below.



Solving by Hand

- Start with source, add it to the **permanent set**.
- Find a node adjacent to the permanent set that is closest to the source, add it to the permanent set.
- Add this node to the permanent set, and note the distance from the source.
- Repeat: add the node that is adjacent to the permanent set, that is closest to the source.
- This methodology gives you the shortest route to *every point!*

Minimal Spanning Tree

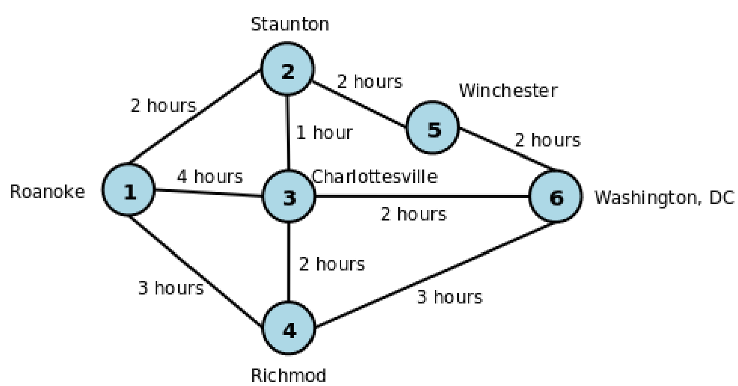
- Objective is to connect all nodes in the network using the least possible distance or cost.
- Examples:
  - Cable company running cable to a series of neighborhoods
  - City planner may design roads to connect various destinations.

Solution Technique by Hand

- Often solving these problems by hand is easier than using a computer.
- Start at any node.
- Select the path to the closest node. At this node and path to the network.
- From the nodes on the final network, select the path to the closest node not yet on the network.
- Repeat until you have reached all nodes.

Example

- Lets use the same example, but suppose the objective is to connect all nodes to an electrical grid, using the least amount of wire possible.
- The possible routes and distances are given below.

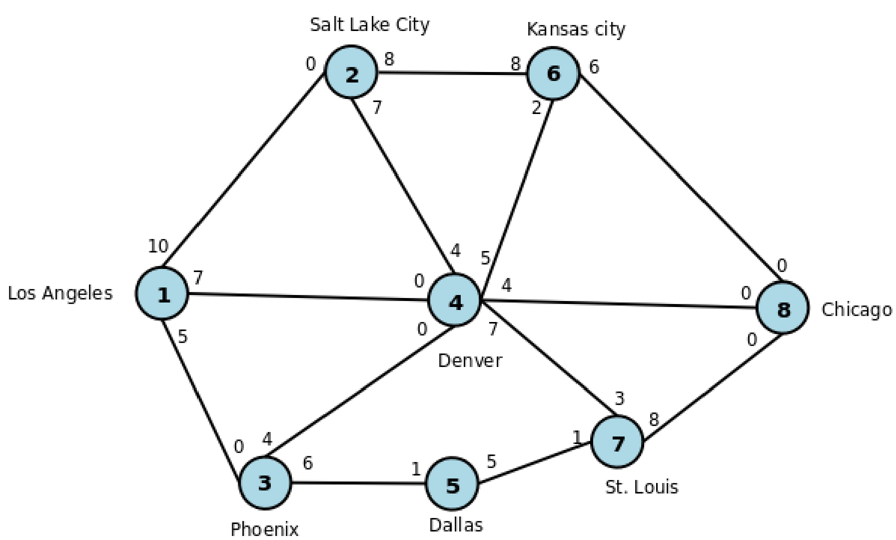


Maximal Flow Problems

- In previous problems: the branches did not have a limited capacity.
- Objective: select a single path that maximizes the total amount of flow from the source to the destination.
- Problem: paths are limited by the amount of flow they will allow.
- Examples:
  - Maximize the amount of flow of oil, gas, or water through pipelines.
  - Maximize the flow of traffic through a road network.
  - Maximize the flow of products through a production line system.
  - Maximize the flow of forms through a paperwork processing system?

Example

Suppose the FAA has just granted a license to a new airline. The airline wants to maximize the number of flights it can send from Los Angeles to Chicago, but for safety reasons, FAA regulations restrict the amount of air traffic between the cities. The flights per day for each route are shown in the following network:



Setting Up Maximal Flow Problems

- Notice traffic can move in *two directions*: each unique path and direction is a decision variable.
- If there are n nodes, there are a maximum of $n^2 + 2$ variables and even more constraints.

- Create two extra variables for the total coming in to the network, and the total leaving the network.
- Each path, in each direction, has its own constraint.
- The total flow of traffic leaving the source must equal the total flow entering destination.
- The total flow entering each intermediate node must equal the total flow leaving the node.

Worksheet questions

1. Make cells for the total flow entering the system, and the total flow leaving the system.
2. Make an n x n matrix of decision variables, for each node going to every other node.
3. Make an n x n matrix of decision constraints, according to your network map.
4. For every row, compute the sum of all flow leaving the node.
   - These are simply the row totals with one exception: the total leaving the destination node includes the 'total leaving' variable.
5. For every column, compute the sum of all flow entering the node.
   - These are simply the column totals with one exception: the total entering the source node includes the 'total entering' variable.
6. Set 'total entering' equal to the total leaving the source node.
7. Set 'total leaving' equal to the total entering the destination node.
8. Objective is to maximize the total leaving the network (or total entering).
9. Constraints:
   (a) Each branch and direction has a limited flow.
   (b) Each branch and direction must be an integer.
   (c) Total entering must equal total leaving.
   (d) The total amount entering each node must equal the total amount leaving.
   (e) Assume non-negativity and linear model.

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Quiz!

A new stadium complex is being constructed, and a city planner is interested in determining whether the city streets from the interstate to the stadium can accommodate the traffic before and after the game. The various traffic arteries along with their capacity of cars (in thousands) over a two hour period is given in the network below. Determine the maximum amount of traffic the city can accommodate. Police officers will direct traffic immediately following the games and must know how much traffic to direct over what routes. For each branch of traffic and for each direction, how many cars should the police officers let through? Fill in your answers in the diagram below. How many total cars can be accommodated?