Transportation Models

BUS 735: Business Decision Making and Research

Goals of this class meeting

- Specific Goals:
  - Learn how to formulate models involving transporting goods from suppliers to destinations.
  - Learn how to use the transportation model framework for finding optimal assignments.
  - Continue to perfect our linear programming / Excel skills!
- Learning Objectives:
  - Be able to construct and solve linear programming models to answer business optimization problems.
  - Be able to use standard computer packages such as SPSS and Excel to conduct the quantitative analyses described in the learning objectives above.

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- **Transportation Models**: class of problems involving transporting goods from suppliers to destinations, usually at minimum cost.
- Assumptions:
  - Each source has a fixed supply (not essential).
  - Each destination has a fixed demand (not essential).
- The cost of transporting goods differs between points.
- Meeting demand at each location typically requires supply from multiple sources.

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Example: Getting grain to mills

- We have three different grain elevators scattered around the Great Plains that can supply grain:
  1. Kansas City supplies 150 tons of grain.
  2. Omaha supplies 175 tons of grain.
  3. Des Moines supplies 275 tons of grain.
- We have three different grain mills that need grain:
  1. Chicago needs 200 tons of grain.
  2. St. Louis needs 100 tons of grain.
  3. Cincinnati needs 300 tons of grain.

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Transportation Costs

Differing distances between locations, and different gasoline prices along the routes, lead to different costs for transportation.

	Destination Cities		
Source Cities	(A) Chicago	(B) St. Louis	(C) Cincinnati
(1) Kansas City	\$6	\$8	\$10
(2) Omaha	\$7	\$11	\$11
(3) Des Moines	\$4	\$5	\$12

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What do we do?

- We want to know how much grain we should send from each grain elevator to each grain mill.
- What is our objective?
- What are our choice variables? How many are there?
- Notation: let x_{1A} denote the amount of grain coming from source 1 (Kansas City) to destination A (Chicago).

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Model Balance

- A **balanced transportation model** is one where total demand is equal to total supply.
  - All supplies will be used $\rightarrow$ supply constraints have $=$.
  - All demands are satisfied $\rightarrow$ demand constraints have $=$.
- If demand exceeds supply (**unbalanced transportation model**):
  - All supplies will be used $\rightarrow$ supply constraints have $=$.
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Assignment Models

- **Assignment models** are like transportation models, except you decide whether or not to assign a "source" to a "destination" (or employee to a task).
- Decision variables are **binary**.
- Suppose you have 3 employees and 3 tasks. How many different possible assignments are there?
- Constraints:
  - Each assignment must get at most 1 assignee.
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  - Non-negativity constraints.
  - Integer constraints (use Integer Programming).

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Homework

- Pages 251-266, problems 13, 14, 21, 48.