# Network Models

BUS 735: Business Decision Making and Research

| Learning Objective | Active Learning Activity |
|---|---|
| Learn how to solve minimum distance network problems | Lecture<br><br>Example solved by hand. |
| Learn how to solve minimal spanning tree problems | Lecture<br><br>Example solved by hand. |
| Learn how to solve maximal flow problems | Lecture<br><br>Example solved with a computer. |
| Assess what we have learned. | Quiz |

- In these problems, there is only one item (person/truck/etc) that comes from one source and arrives one destination.
- The problem: there are lots of intermediate points (**nodes**) and lots of routes to choose from.
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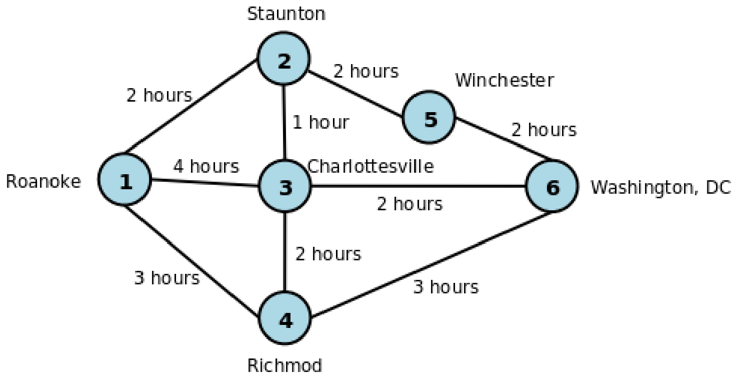
- Goal: Drive from Roanoke, VA to Washington DC, using the fastest route (without using Mapquest).
- The possible routes and distances are given below.



- Start with source, add it to the **permanent set**.
- Find a node adjacent to the permanent set that is closest to the source, add it to the permanent set.
- Add this node to the permanent set, and note the distance from the source.
- Repeat: add the node that is adjacent to the permanent set, that is closest to the source.
- This methodology gives you the shortest route to *every point!*

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- Examples:
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- Often solving these problems by hand is easier than using a computer.
- Start at any node.
- Select the path to the closest node. At this node and path to the network.
- From the nodes on the final network, select the path to the closest node not yet on the network.
- Repeat until you have reached all nodes.

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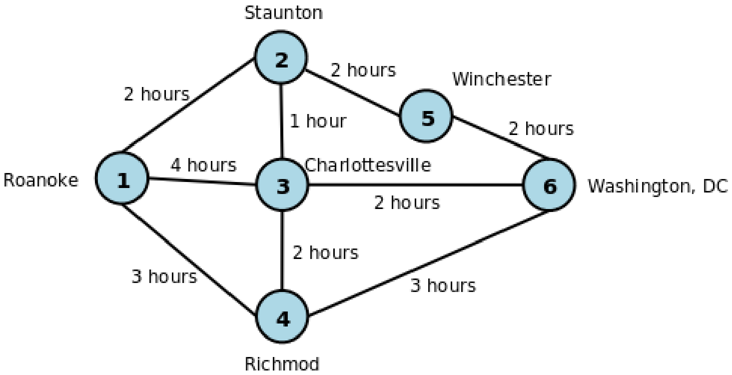

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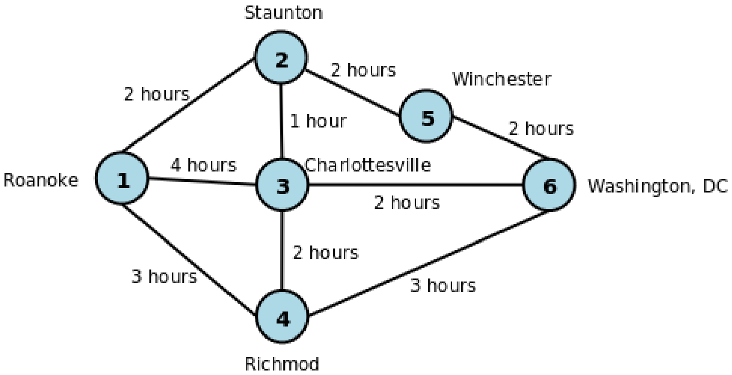

- In previous problems: the branches did not have a limited capacity.
- Objective: select a single path that maximizes the total amount of flow from the source to the destination.
- Problem: paths are limited by the amount of flow they will allow.
- Examples:
    - Maximize the amount of flow of oil, gas, or water through pipelines.
    - Maximize the flow of traffic through a road network.
    - Maximize the flow of products through a production line system.
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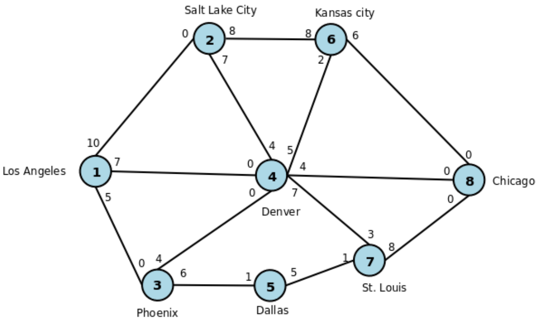
Suppose the FAA has just granted a license to a new airline. The airline wants to maximize the number of flights it can send from Los Angeles to Chicago, but for safety reasons, FAA regulations restrict the amount of air traffic between the cities. The flights per day for each route are shown in the following network:



- Notice traffic can move in *two directions*: each unique path and direction is a decision variable.
- If there are $n$ nodes, there are a maximum of $n^2 + 2$ variables and even more constraints.
- Create two extra variables for the total coming in to the network, and the total leaving the network.
- Each path, in each direction, has its own constraint.
- The total flow of traffic leaving the source must equal the total flow entering destination.
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