# Transportation Models

BUS 735: Business Decision Making and Research

# Goals of this class meeting



- Specific Goals:
  - Learn how to formulate models involving transporting goods from suppliers to destinations.
  - Learn how to use the transportation model framework for finding optimal assignments.
  - Continue to perfect our linear programming / Excel skills!
- Learning Objectives:
  - Be able to construct and solve linear programming models to answer business optimization problems.
  - Be able to use standard computer packages such as Excel to conduct the quantitative analyses described in the learning objectives above.

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# Reading



Taylor, Chapter 6
Pages: 237-244, 251-253

# Transportation Models



- **Transportation Models**: class of problems involving transporting goods from suppliers to destinations, usually at minimum cost.
- Assumptions:
  - Each source has a fixed supply (not essential).
  - Each destination has a fixed demand (not essential).
- The cost of transporting goods differs between points.
- Meeting demand at each location typically requires supply from multiple sources.

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# Example: Getting grain to mills



- We have three different grain elevators scattered around the Great Plains that can supply grain:
  1. Kansas City supplies 150 tons of grain.
  2. Omaha supplies 175 tons of grain.
  3. Des Moines supplies 275 tons of grain.
- We have three different grain mills that need grain:
  1. Chicago needs 200 tons of grain.
  2. St. Louis needs 100 tons of grain.
  3. Cincinnati needs 300 tons of grain.

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# Transportation Costs



Differing distances between locations, and different gasoline prices along the routes, lead to different costs for transportation.

| | Destination Cities | | |
|---|---|---|---|
| **Source Cities** | (A) Chicago | (B) St. Louis | (C) Cincinnati |
| (1) Kansas City | \$6 | \$8 | \$10 |
| (2) Omaha | \$7 | \$11 | \$11 |
| (3) Des Moines | \$4 | \$5 | \$12 |

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# What do we do?



- We want to know how much grain we should send from each grain elevator to each grain mill.
- What is our objective?
- What are our choice variables? How many are there?
- Notation: let $x_{1A}$ denote the amount of grain coming from source 1 (Kansas City) to destination A (Chicago).

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# Model Balance



- A **balanced transportation model** is one where total demand is equal to total supply.
  - All supplies will be used $\rightarrow$ supply constraints have $=$.
  - All demands are satisfied $\rightarrow$ demand constraints have $=$.
- If demand exceeds supply (**unbalanced transportation model**):
  - All supplies will be used $\rightarrow$ supply constraints have $=$.
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# Assignment Models



- **Assignment models** are like transportation models, except you decide whether or not to assign a "source" to a "destination" (or employee to a task).
- Decision variables are **binary**.
- Suppose you have 3 employees and 3 tasks. How many different possible assignments are there?
- Constraints:
  - Each assignment must get at most 1 assignee.
  - Each assignee must get at most 1 assignment.
  - Non-negativity constraints.
  - Integer constraints (use Integer Programming).

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# Homework



Chapter 6 Problems #26, #32, #37