Measuring Interest Rates

Economics 301: Money and Banking

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1.1 Goals

Goals and Learning Outcomes

- Goals:
  - Learn to compute present values, rates of return, rates of return.
- Learning Outcomes:
  - LO3: Predict changes in interest rates using fundamental economic theories including present value calculations, behavior towards risk, and supply and demand models of money and bond markets.

1.2 Reading

Reading

- Read Hubbard and O'Brien, Chapter 3.

2 Measuring Present Value

2.1 Simple Loans

Cash Flows

- **Cash flows:** size and timing of payments made for various debt instruments.
- **Present value:** aka **present discounted value**, discounts payments made in the future to a current date equivalent.
- Present value depends on assumption for interest rate.
  - Higher interest rates - higher degree of discount.

Simple Loan Example

- Simple loan: lender provides funds to borrower, borrower pays back principal and interest at maturity date.
- Suppose interest rate is 5% (denote with i), simple loan of \$100 (denote with P).
- Balance (denote with A) with a one year maturity:
  - $A_1 = P(1+i) = \$100(1+0.05) = \$105$.
- Let it ride for another year...
  - $A_2 = A_1(1+i) = \$105(1+0.05) = \$110.25$
  - $A_2 = P(1+i)(1+i) = P(1+i)^2 = \$100(1+0.05)^2 = \$110.25$
- At the end of n years, we have
  - $A_n = P(1+i)^n$.

Present Value

- Present value: indifferent between \$100 today, \$105 next year, or \$110.25 in two years.
- Given future cash flow of \$105 or \$110.25, respectively, the present value is,

$$PV = 100 = \frac{105}{(1+0.05)}$$

$$PV = 100 = \frac{110.25}{(1+0.05)^2}$$

- General formula,

$$PV = \frac{CF_n}{(1+i)^n}$$

- Example: what is the present value of \$100,000 to be paid in 30 years if the interest rate is 4%?

2.2 Other Debt Instruments

Types of Credit Market Instruments

- Simple loan.
- Fixed-payment loan: borrower makes a fixed payment (that includes interest and principal) each period until maturity date.

- Coupon bond: borrower pays fixed interest payments (coupon payments) until maturity date, pays face value at maturity.
  - **Coupon rate:** dollar amount of coupon payments as a percentage of face value. Related to, but not exactly an interest rate.
- Discount bond: bought at a price below its face value, makes no payments until maturity date, at which time pays face value.

Compounded Interest

- **Compounded interest:** when interest payments are made multiple times in a given period.
- Compounded annually: full interest payment paid out once per year.
- Compounded quarterly: payment for 1/4 of interest rate made 4 times per year.
- Compounded monthly: payment for 1/12 of interest rate made 12 times per year.
- Compounded daily: payment for 1/365 of interest rate made 365 times per year.
- Compounded continuously: interest payments constantly made. Occurs in nature.

Present Value Computations

- Present value of a stream of cash flows (CF_t) from time $t = 0$ (today) to $t = T$:

$$PV = \sum_{t=0}^{T} \frac{CF_t}{(1+i)^t} = CF_0 + \frac{CF_1}{1+i} + \frac{CF_2}{(1+i)^2} + ... + \frac{CF_T}{(1+i)^T}$$

- Suppose you have an auto loan,
  - Annual interest rate is 6% interest.
  - Compounded monthly.
  - Five year loan.
  - Your monthly payment is $200.
  - How much was your car?

Present Value Computations

- The geometric series is a useful mathematical tool in PV computations: If $\beta \in (0, 1)$, then,

$$\frac{1}{1-\beta} = 1 + \beta + \beta^2 + \beta^3 + \beta^4 + ...$$

- Used in present values: $\beta = \frac{1}{1+i}$ which is between 0 and 1 for positive interest rates.
- Used for cash flows that occur every period forever. Eg: Perpetuity, stock dividends?

Present Value Calculations

- Multiply present value $1/(1-\beta)$ (previous slide) by β^T:

$$\frac{\beta^T}{1-\beta} = \beta^T + \beta^{(T+1)} + \beta^{(T+2)} + \beta^{(T+3)} + ...$$

Subtract the this equation from $1/(1-\beta)$ (previous slide):

$$\frac{1-\beta^T}{1-\beta} = 1 + \beta + \beta^2 + \beta^3 + ... + \beta^{T-1}$$

Used for cash flows that begin in current period (0) through period T-1

- For cash flows beginning in period s and lasting through period T:

$$\frac{\beta^s - \beta^{T+1}}{1-\beta} = \beta^s + \beta^{s+1} + \beta^{s+2} + ... + \beta^T$$

More Computations

- Compute the present value of coupon bond with
  - Face value $3000.
  - 10 year maturity.
  - Coupon rate 6%.
  - Annual payment beginning in one year.
  - Prevailing interest rate 5%.
- Compute the present value of a discount bond with,
  - Face value $5000.
  - 5 year maturity.
  - Prevailing interest rate 8%.

3 Measuring Return

3.1 Yield to Maturity

Yield to Maturity

- **Yield to maturity:** the annual interest rate that equates the present value of cash flow of payments received from a debt instrument with its current day value.
- Example: yield to maturity for a simple loan.
  - PV = Cash borrowed = \$200.
  - CF = Cash flow = payment received after $n = 5$ years \$280.51.

$$PV = \frac{CF}{(1+i)^n} \quad \rightarrow \quad 200 = \frac{280.51}{(1+i)^5}$$

$$(1+i)^5 = \frac{280.51}{200} \quad \rightarrow \quad 1+i = \left(\frac{280.51}{200}\right)^{\frac{1}{5}}$$

$$1+i = 1.07 \quad \rightarrow \quad i = 7\%$$

Yield to Maturity: Coupon bond

- Present value of a coupon bond for,
  - Coupon payment = CF.
  - Face value = F.
  - Years to maturity = T.

$$PV = \frac{CF}{(1+i)} + \frac{CF}{(1+i)^2} + ... + \frac{CF}{(1+i)^T} + \frac{F}{(1+i)^T}$$

$$PV = \sum_{t=1}^{T} \frac{CF}{(1+i)^t} + \frac{F}{(1+i)^T}$$

- To find yield to maturity, solve for i. Impossible to do algebraically → use financial calculator.

3.2 Rate of Return

Rate of Return

- **Rate of return:** the total benefits received from holding a security, expressed as a percentage of purchase price.
- Rate of return includes interest payments *plus capital gains.*

- Rate of return for holding a bond from time t to $t+1$ is,

$$R = \frac{CF + P_{t+1} - P_t}{P_t}$$

  - R: rate of return.
  - P_t: price of bond at time t.
- Can also express rate of return as the sum, $R = i + g$, where,

$$\text{rate of capital gain} = g = \frac{P_{t+1} - P_t}{P_t},$$

$$\text{interest rate} = i = \frac{CF}{P_t}$$

Rate of Return

- Suppose a debt instrument is held for one year that is,
  - purchased for $1,500,
  - makes a single interest payment of $100,
  - sold for $1,600.
- What is the interest rate, rate of capital gain, rate of return?
- Suppose instead the sale price is $1,400. What is the interest rate, rate of capital gain, rate of return?

3.3 Maturity, Volatility, and Return

Maturity, Volatility, and Return

- Long-term debt instruments have a high degree of interest rate risk.
- **interest rate risk**: changes in interest rates over the life of the debt instrument influence the secondary market price of the bond, influencing capital gains and therefore rate of return.
- Prices and returns for long-term bonds are *more volatile* than short-term bonds.
- Interest payments are therefore typically higher for long-term bonds.

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4.1 Coming up next...

Coming up next...

- Analyzing behavior of interest rates and asset markets using supply and demand model.
- Reading: Chapter 4.