

Measuring Interest Rates

Economics 301: Money and Banking

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1.1 Goals

Goals and Learning Outcomes

- Goals:
 - Learn to compute present values, rates of return, rates of return.
- Learning Outcomes:
 - LO3: Predict changes in interest rates using fundamental economic theories including present value calculations, behavior towards risk, and supply and demand models of money and bond markets.

1.2 Reading

Reading

- Read Hubbard and O'Brien, Chapter 3.

2 Measuring Present Value

2.1 Simple Loans

Cash Flows

- **Cash flows:** size and timing of payments made for various debt instruments.
- **Present value:** aka **present discounted value**, discounts payments made in the future to a current date equivalent.
- Present value depends on assumption for interest rate.
 - Higher interest rates - higher degree of discount.

Simple Loan Example

- Simple loan: lender provides funds to borrower, borrower pays back principal and interest at maturity date.
- Suppose interest rate is 5% (denote with i), simple loan of \$100 (denote with P).
- Balance (denote with A) with a one year maturity:
 - $A_1 = P(1 + i) = \$100(1 + 0.05) = \105 .
- Let it ride for another year...
 - $A_2 = A_1(1 + i) = \$105(1 + 0.05) = \110.25
 - $A_2 = P(1 + i)(1 + i) = P(1 + i)^2 = \$100(1 + 0.05)^2 = \$110.25$
- At the end of n years, we have
 - $A_n = P(1 + i)^n$.

Present Value

- Present value: indifferent between \$100 today, \$105 next year, or \$110.25 in two years.
- Given future cash flow of \$105 or \$110.25, respectively, the present value is,

$$PV = 100 = \frac{105}{(1 + 0.05)}$$

$$PV = 100 = \frac{110.25}{(1 + 0.05)^2}$$

- General formula,

$$PV = \frac{CF_n}{(1 + i)^n}$$

- Example: what is the present value of \$100,000 to be paid in 30 years if the interest rate is 4%?

2.2 Other Debt Instruments

Types of Credit Market Instruments

- Simple loan.
- Fixed-payment loan: borrower makes a fixed payment (that includes interest and principal) each period until maturity date.

- Coupon bond: borrower pays fixed interest payments (coupon payments) until maturity date, pays face value at maturity.
 - **Coupon rate:** dollar amount of coupon payments as a percentage of face value. Related to, but not exactly an interest rate.
- Discount bond: bought at a price below its face value, makes no payments until maturity date, at which time pays face value.

Compounded Interest

- **Compounded interest:** when interest payments are made multiple times in a given period.
- Compounded annually: full interest payment paid out once per year.
- Compounded quarterly: payment for 1/4 of interest rate made 4 times per year.
- Compounded monthly: payment for 1/12 of interest rate made 12 times per year.
- Compounded daily: payment for 1/365 of interest rate made 365 times per year.
- Compounded continuously: interest payments constantly made. Occurs in nature.

Present Value Computations

- Present value of a stream of cash flows (CF_t) from time $t = 0$ (today) to $t = T$:

$$PV = \sum_{t=0}^T \frac{CF_t}{(1+i)^t} = CF_0 + \frac{CF_1}{1+i} + \frac{CF_2}{(1+i)^2} + \dots + \frac{CF_T}{(1+i)^T}$$

- Suppose you have an auto loan,
 - Annual interest rate is 6% interest.
 - Compounded monthly.
 - Five year loan.
 - Your monthly payment is \$200.
 - How much was your car?

Present Value Computations

- The geometric series is a useful mathematical tool in PV computations:
If $\beta \in (0, 1)$, then,

$$\frac{1}{1 - \beta} = 1 + \beta + \beta^2 + \beta^3 + \beta^4 + \dots$$

- Extensions:

$$\frac{\beta^{(T+1)}}{1 - \beta} = \beta^{(T+1)} + \beta^{(T+2)} + \beta^{(T+3)} + \beta^{(T+4)} + \dots$$

Subtract the second equation from the first,

$$\frac{1 - \beta^{T+1}}{1 - \beta} = 1 + \beta + \beta^2 + \beta^3 + \dots + \beta^T$$

- Used in present values: $\beta = \frac{1}{1+i}$ which is between 0 and 1 for positive interest rates.

More Computations

- Compute the present value of coupon bond with
 - Face value \$3000.
 - 10 year maturity.
 - Coupon rate 5%.
 - Prevailing interest rate in economy 5%.
- Compute the present value of a discount bond with,
 - Face value \$5000.
 - 5 year maturity.
 - Prevailing interest rate in economy 8%.

3 Measuring Return

3.1 Yield to Maturity

Yield to Maturity

- **Yield to maturity:** the annual interest rate that equates the present value of cash flow of payments received from a debt instrument with its current day value.
- Example: yield to maturity for a simple loan.

- PV = Cash borrowed = \$200.
- CF = Cash flow = payment received after $n = 5$ years \$280.51.

$$PV = \frac{CF}{(1+i)^n} \rightarrow 200 = \frac{280.51}{(1+i)^5}$$

$$(1+i)^5 = \frac{280.51}{200} \rightarrow 1+i = \left(\frac{280.51}{200}\right)^{\frac{1}{5}}$$

$$1+i = 1.07 \rightarrow i = 7\%$$

Yield to Maturity: Coupon bond

- Present value of a coupon bond for,
 - Coupon payment = CF .
 - Face value = F .
 - Years to maturity = T .

$$PV = \frac{CF}{(1+i)} + \frac{CF}{(1+i)^2} + \dots + \frac{CF}{(1+i)^T} + \frac{F}{(1+i)^T}$$

$$PV = \sum_{t=1}^T \frac{CF}{(1+i)^t} + \frac{F}{(1+i)^T}$$

- To find yield to maturity, solve for i . Impossible to do algebraically \rightarrow use financial calculator.

3.2 Rate of Return

Rate of Return

- **Rate of return:** the total benefits received from holding a security, expressed as a percentage of purchase price.
- Rate of return includes interest payments *plus capital gains*.
- Rate of return for holding a bond from time t to $t + 1$ is,

$$R = \frac{CF + P_{t+1} - P_t}{P_t}$$

- R : rate of return.
- P_t : price of bond at time t .
- Can also express rate of return as the sum, $R = i + g$, where,

$$\text{rate of capital gain} = g = \frac{P_{t+1} - P_t}{P_t},$$

$$\text{interest rate} = i = \frac{CF}{P_t}$$

Rate of Return

- Suppose a debt instrument is held for one year that is,
 - purchased for \$1,500,
 - makes a single interest payment of \$100,
 - sold for \$1,600.
- What is the interest rate, rate of capital gain, rate of return?
- Suppose instead the sale price is \$1,400. What is the interest rate, rate of capital gain, rate of return?

3.3 Maturity, Volatility, and Return

Maturity, Volatility, and Return

- Long-term debt instruments have a high degree of interest rate risk.
- **interest rate risk:** changes in interest rates over the life of the debt instrument influence the secondary market price of the bond, influencing capital gains and therefore rate of return.
- Prices and returns for long-term bonds are *more volatile* than short-term bonds.
- Interest payments are therefore typically higher for long-term bonds.

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4.1 Coming up next...

Coming up next...

- Homework #2: Interest rates. Posted on the class website.
- Analyzing behavior of interest rates and asset markets using supply and demand model.
- Reading: Chapter 4.