Economic Growth: Solow Growth Model

ECO 305: Intermediate Macroeconomics

Goals Reading and Exercises

Use Solow growth model theory to explain...

- Why some countries have high rates of growth and other have low rates of growth
- What factors affect economic development and growth
- Shortcomings of the theory

Goals Reading and Exercises

- Williamson, Chapter 7, pp. 249-255: Solow Model
- Williamson, Chapter 7, pp. 255-264: Long-run effects from changes to savings, technology, depreciation, and population growth
- Canvas Quiz due Wednesday 11:59 PM. Multiple-choice, 10 questions, unlimited attempts allowed, only best score counts
- Homework/Exercise due Friday 11:59 PM. We will work together in class on Thursday

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Over time Across Countries Covariates

Economic Growth Facts Over Time

- Before the industrial revolution in about 1800, standards of living did not grow much over time.
- Since the industrial revolution, per-capita income growth has grown slowly and steadily in the richest countries
 - The average growth rate of per-capita income in the richest countries is about 1-3%.

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Over time Across Countries Covariates

Economic Growth Facts Across Countries

- Before the industrial revolution, standards of living were similar across much of the world.
- Differences in per-capita income across countries have grown significantly since the industrial revolution.
- Rich countries today are similar in terms of per-capita income growth.
- Lesser-developed countries today are less alike in terms of per-capita income growth.

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Over time Across Countries Covariates

Richest Economies (Real GDP Per Capita in 2020) 5/20



Over time Across Countries Covariates

Poorest Economies (Real GDP Per Capita in 2020) 6/20



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Over time Across Countries Covariates

Fastest Growing Economies (1999-2019)

7/20



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Over time Across Countries Covariates

Slowest Growing Economies (1999-2019)



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Over time Across Countries Covariates

Growth Covariates

- There is a negative relationship between population growth rates and per-capita income growth rates.
- There is a positive relationship between investment rate (as a percentage of real GDP) and per-capita income for **lesser** developed countries.
- There is a negative relationship between investment rate and per-capita income for **more developed countries**.

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Mathematical Foundation Predictions for Growth Convergence of Lesser-Developed and Developed Economies Solving for Steady State

Foundations of Solow Growth Model

10/ 20

Consumer Behavior

- Private consumer savings: S = sY
- Population growth: N' = (1 + n)N
- $s \in (0,1)$: exogenous savings rate
- n ∈ (0, 1): exogenous population growth rate

Government Behavior

- Government budget: G = T + B
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- Let s_N = s − b, s_N ∈ (0,1) denote national savings rate

Investment and Capital

- Evolution of Capital Stock: K' = (1 - d)K + I
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- I: real quantity of investment
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- Production function: Y = zf(K, N)
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Further Assumptions

Diminishing marginal of capital

• Constant returns to scale: When an economy increases all of its factors of production (i.e. both labor and capital) by the same percentage, production goes up by the same percentage

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Results of the Solow Growth Model

12/ 20

Conclusions

Evolution of capital stock per worker:

$$k' = \left(\frac{1-d}{1+n}\right)k + \left(\frac{s_N z}{1+n}\right)f(k)$$

Savings and Investment:

• I = S - B

•
$$I = sY - bY = (s - b)Y = s_NY$$

• Investment = National Savings



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$$k' = \left(\frac{1-d}{1+n}\right)k + \left(\frac{s_N z}{1+n}\right)f(k)$$

Savings and Investment:

● *I* = *S* − *B*

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$$I = sY - bY = (s - b)Y = s_NY$$

• Investment = National Savings



Mathematical Foundation Predictions for Growth Convergence of Lesser-Developed and Developed Economic Solving for Steady State

Results of the Solow Growth Model

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Mathematical Foundation Predictions for Growth Convergence of Lesser-Developed and Developed Economie: Solving for Steady State

Economic Growth in Solow Model



Economic Growth

- Increases in real GDP per capita higher at lower levels of capital stock per worker / lower levels of real GDP per capita
- Due to diminishing marginal product of capital

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Mathematical Foundation Predictions for Growth Convergence of Lesser-Developed and Developed Economies Solving for Steady State

Lesser-Developed vs Develped Economies

Model Predictions

- Lesser-developed economies: Higher rates of growth in capital per worker and real GDP
- Developed economies: Slow rates of growth
- Once at steady state (k*, y*), only economic growth would come from shift in z
- Convergence: Lesser-developed economies eventually catch up to highly-developed economies



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Mathematical Foundation Predictions for Growth Convergence of Lesser-Developed and Developed Economies Solving for Steady State

Solving for Steady State







Mathematical Foundation Predictions for Growth Convergence of Lesser-Developed and Developed Economies Solving for Steady State

Solving for Steady State

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Mathematical Foundation Predictions for Growth Convergence of Lesser-Developed and Developed Economies Solving for Steady State

Solving for Steady State

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Solving for Steady State Use the following equation, and set $k' = k = k^*$ $k' = \left(\frac{1-d}{1+n}\right)k + \left(\frac{s_N z}{1+n}\right)f(k)$ $k^* = \left(\frac{1-d}{1+n}\right)k^* + \left(\frac{s_N z}{1+n}\right)f(k^*)$ $(n+d)k^* = s_N z f(k^*)$





Mathematical Foundation Predictions for Growth Convergence of Lesser-Developed and Developed Economies Solving for Steady State

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Solving for Steady State

k

Solving for Steady State

(n+d)k

s_N z f(k)

k





Total Factor Productivity

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Improvement in Total Factor Productivity

- Increase in z causes production function and s_Nzf(k*) to pivot upward
- Result: Higher k* and y*, i.e. higher steady state level of capital per worker, higher real GDP per capita

Total Factor Productivity

Output Per-Worker





ECO 305: Intermediate Macroeconomics Economic Gr

Total Factor Productivity National Savings Rate Population Growth Rate Depreciation Rate

Total Factor Productivity

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Economic Growth: Solow Growth Model

Total Factor Productivity National Savings Rate Population Growth Rate Depreciation Rate

Total Factor Productivity

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Total Factor Productivity National Savings Rate Population Growth Rate Depreciation Rate

National Savings Rate

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Increase in national savings rate

Increase in s_N causes s_Nzf(k*) to pivot upward, but no change in production function
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Total Factor Productivity National Savings Rate Population Growth Rate Depreciation Rate

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Total Factor Productivity National Savings Rate **Population Growth Rate** Depreciation Rate

Population Growth Rate

Increase in Population Growth Rate

- Increase in *n* causes (n + d)k line to pivot upward
- Result: Lower k* and y*, i.e. lower steady state level of capital per worker, lower real GDP per capita



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Total Factor Productivity National Savings Rate Population Growth Rate Depreciation Rate

Depreciation Rate

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Increase in Depreciation Rate of Capital

- Increase in d causes (n + d)k line to pivot upward
- Result: Lower k^* and y^* , i.e. lower steady state level of capital per worker, lower real GDP per capita



Total Factor Productivity National Savings Rate Population Growth Rate Depreciation Rate

Depreciation Rate

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Increase in Depreciation Rate of Capital

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Model Shortcomings

• Not all lesser-developed countries have high rates of growth

- Not all lesser-developed countries are catching up
- Increasing returns to scale, increasing marginal product of capital, possible for lesser-developed economies
- Fails to account for human capital in economic growth

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